Compact representation of distances in a graph : a tour around 2-hop labelings

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Joint work with Siddharth Gupta (Univ. of Warwick),
Adrian Kosowski (NavAlgo)
and Przemysław Uznański (NavAlgo – Univ. of Wrocław)



Encoding distances in a graph

We are given a (weighted) (di-) graph $G = (V, E)$ with n nodes and m edges.

Make any useful pre-computation to answer efficiently online distance queries : what is distance $d(u_1, v_1)$?, $d(u_2, v_2)$?,$d(u_3, v_3)$?,...

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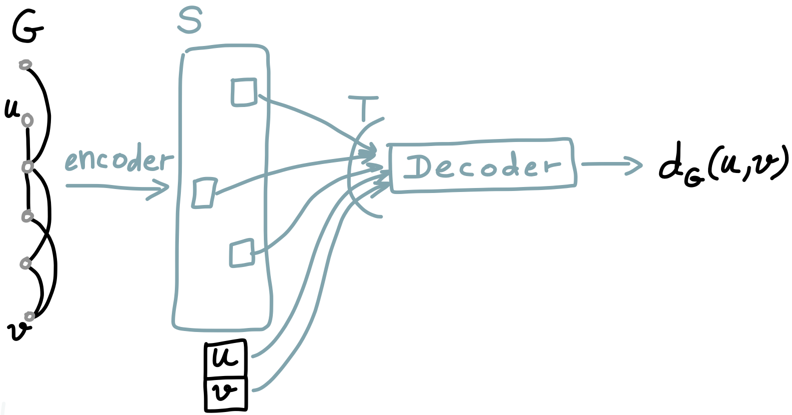
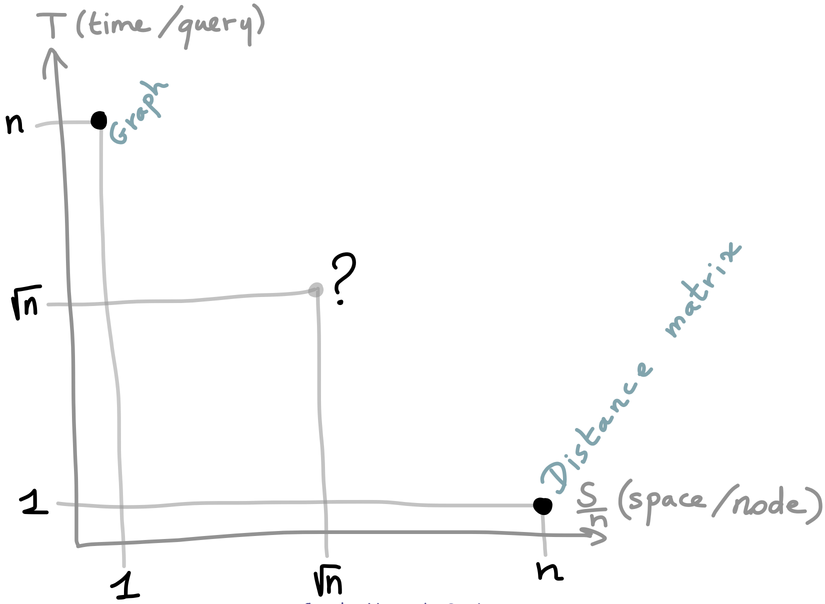
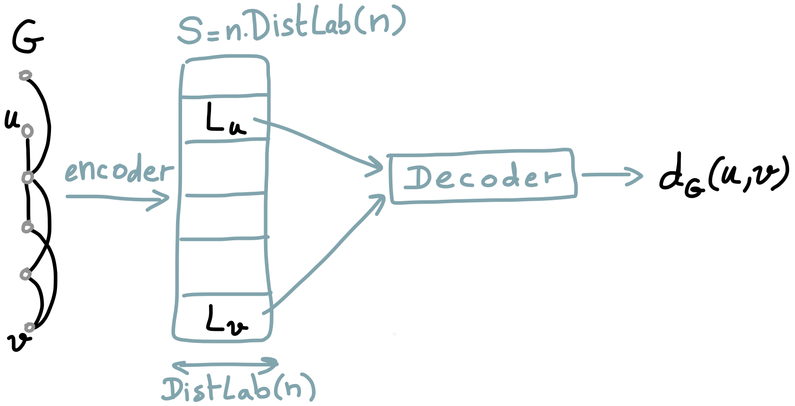
Encoding a graph metric : distance oracles



Size S vs query time T tradeoff (sparse graphs, i.e. $m = O(n)$)



Encoding a graph metric : distance labelings



Encoding a graph metric : 2-hop labelings

A 2-hop labelings is a very simple kind of distance labeling.

The main idea is to associate a set $H_u \subseteq V$ of "hubs" to each node u and to store the distances $d(u, v)$ for all $v \in H_u$.

Also known as hub labeling, or landmark labeling.

One of the two main bulding blocks of classical distance labelings (the other being tree labelings).

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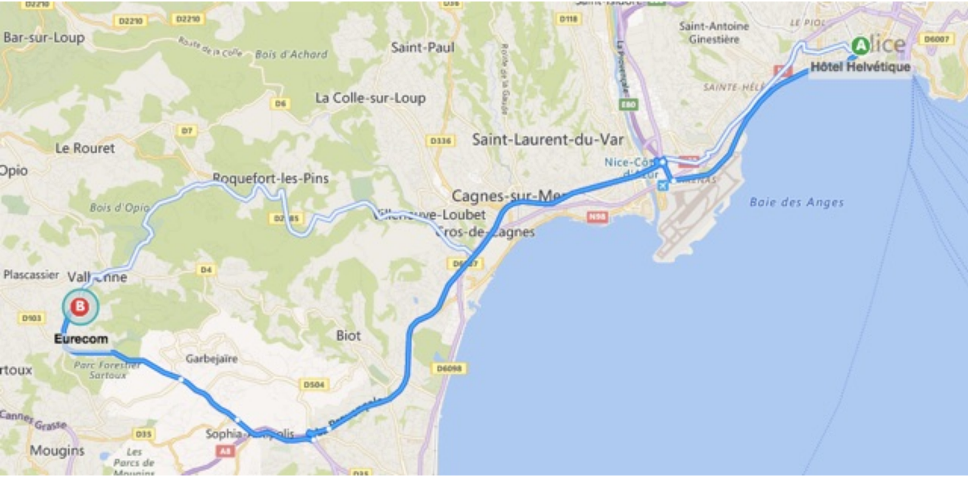

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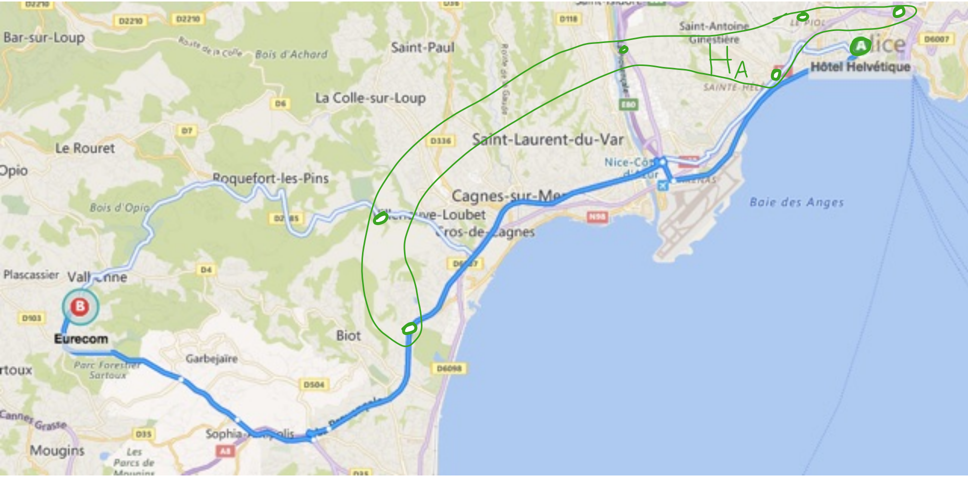

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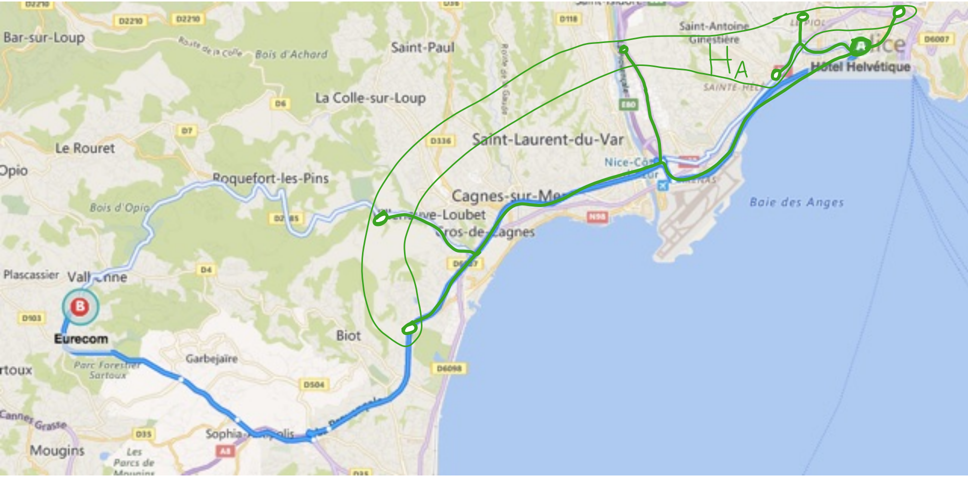

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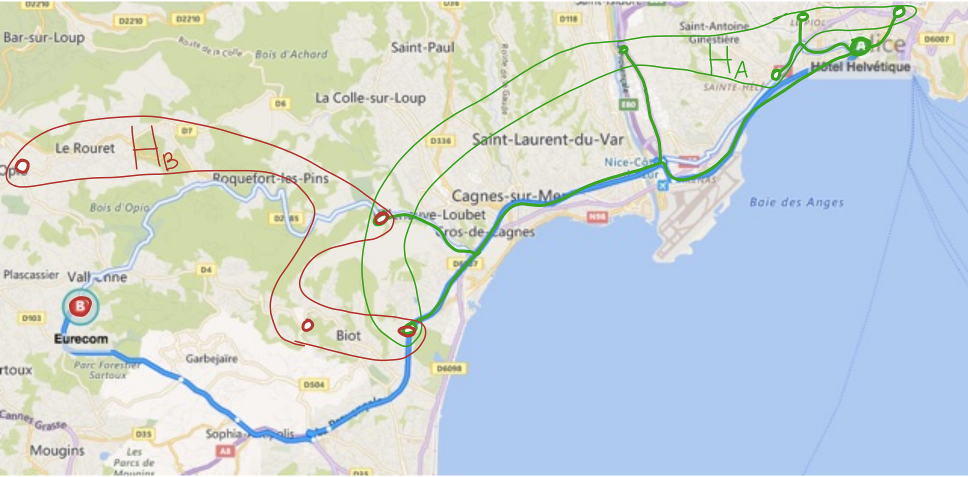

2-hop labeling

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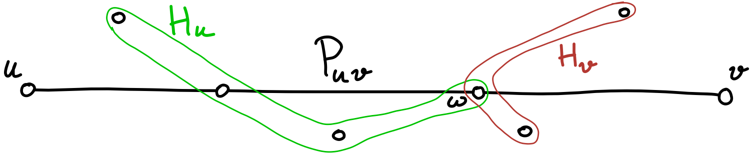
Hub sets

Covering property :

A collection of hub sets $H_u \subseteq V$ for all $u \in V$ is said to cover graph G if for all u, v, there exists $w \in H_u \cap H_v$ with $w \in P_{uv}$, where P_{uv} is a shortest uv-path.



Distance labels : $L_u = \{(w, d(u, w)) : w \in H_u\}$

Distance query : $\text{Dist}\ (L_u, L_v) = \min_{w \in H_u \cap H_v} d(u, w) + d(w, v)$

Introduced by [Gavoille et al. '04 ; Cohen et al. 2003], applied to road networks [Abraham et al. 2010-2013], and other practical networks [Akiba et al. 2013]. Approximability results : [Babenko et al. 2013, Angelidakis et al. 2017].

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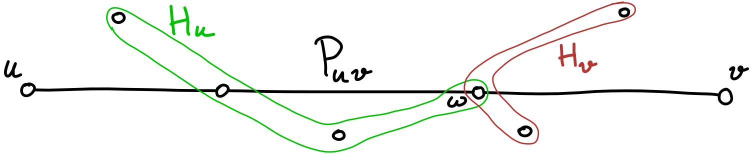

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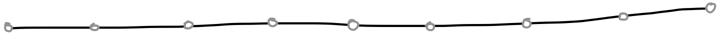
Hub sets covering a path



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A similar construction works for trees, bounded-treewidth graphs and planar graphs.

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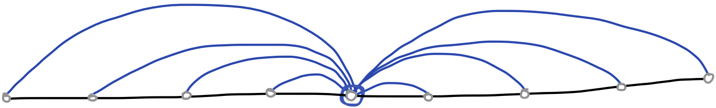

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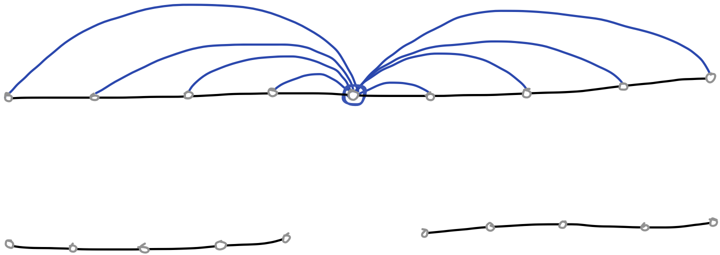

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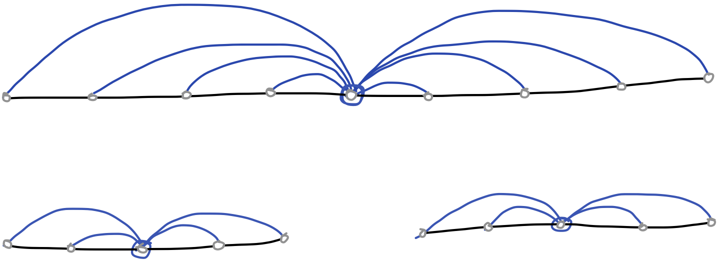

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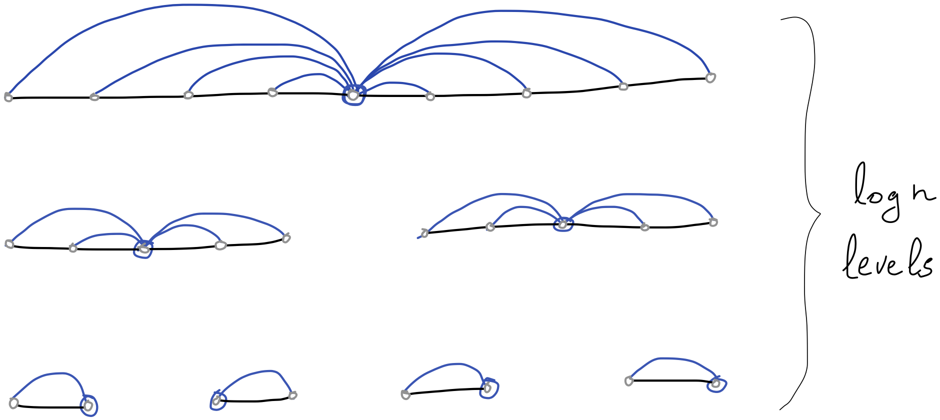

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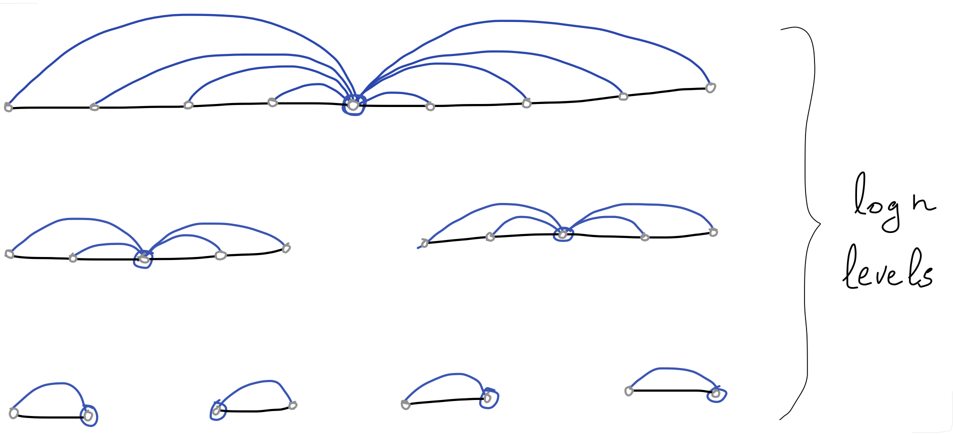

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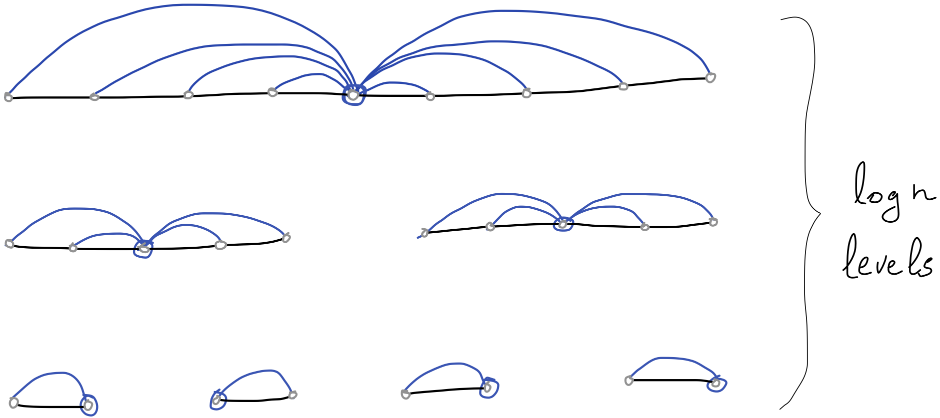

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This talk is about

What graphs do have small hubsets?

No hope for dense graphs :

- average hub-set size is at least $\frac{m}{n}$ as :
- for each edge $uv \in E$, we must have $u \in H_v$ or $v \in H_u$.

Planar graphs have covering hub sets of size $O(\sqrt{n})$, with a best known lower bound of $\Omega(n^{1/3})$ (unweighted). [Gavoille, Peleg, Pérennes, Raz '04].

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Part I : Do practical graphs have small covering hub sets?

Yes! practical graphs tend to have small covering hub sets. [Akiba et al. '13] [Delling et al. 14]

What kind of property they have enables that?

Small highway dimension. [Abraham, Fiat, Goldberg, Werneck '10-13]

More generally, small skeleton dimension. [Kosowski, V., SODA'17]

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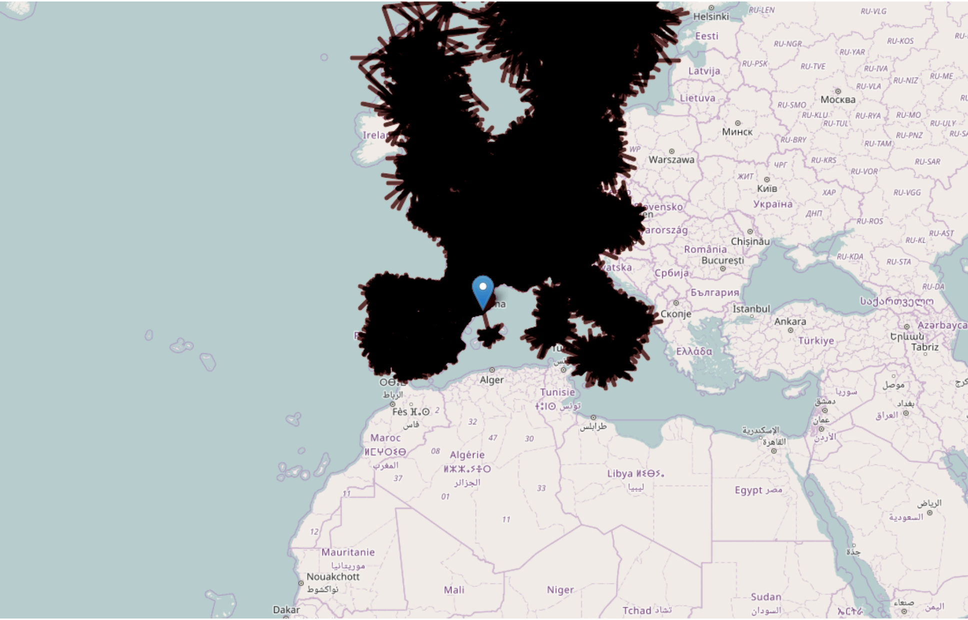
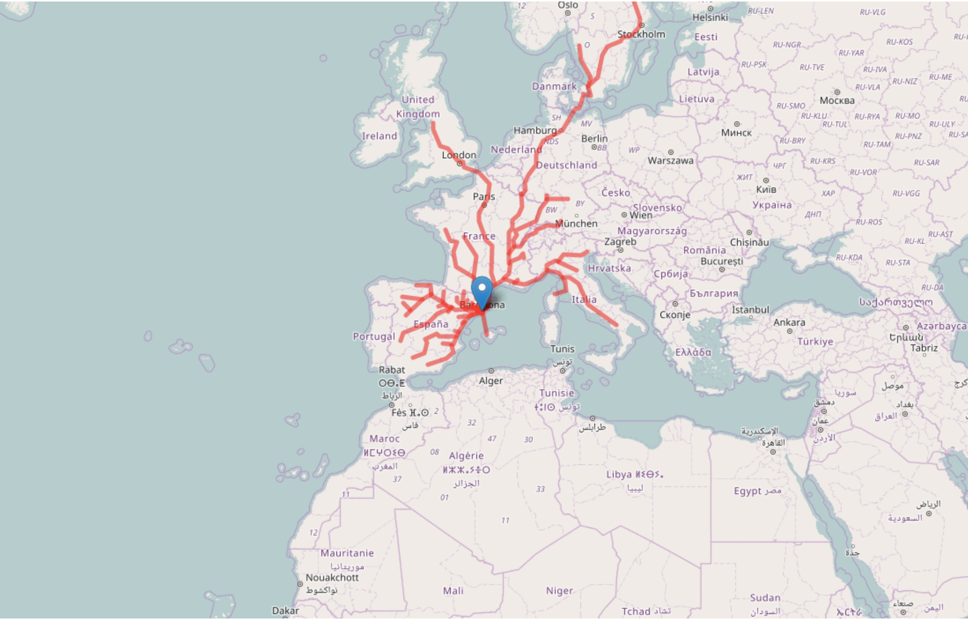
Skeleton dimension

The skeleton dimension k of G is the maximum “width” of a “pruned” shortest path tree.

Barcelona shortest path tree



Barcelona tree skeleton : prune last third



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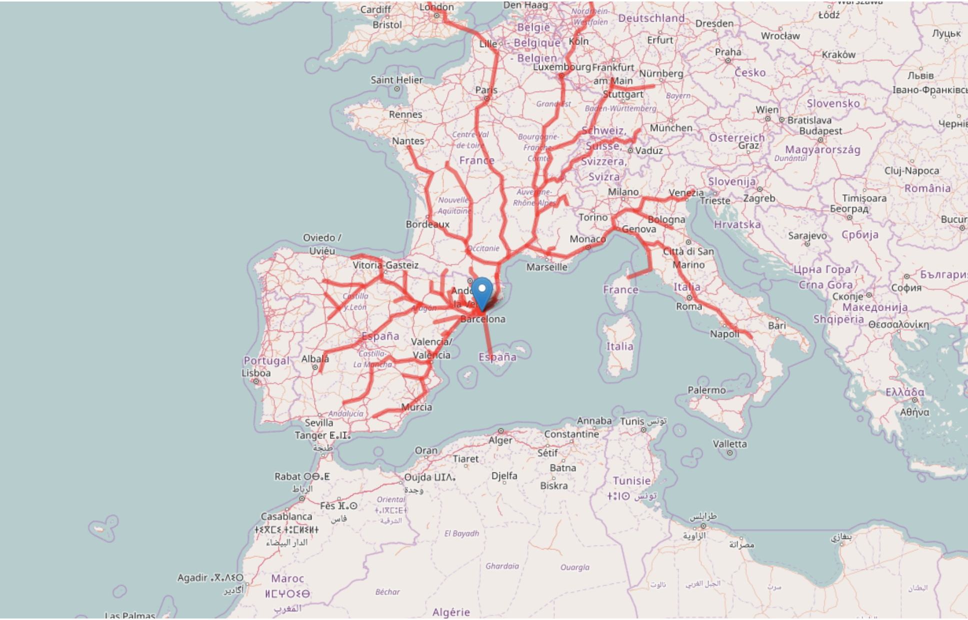

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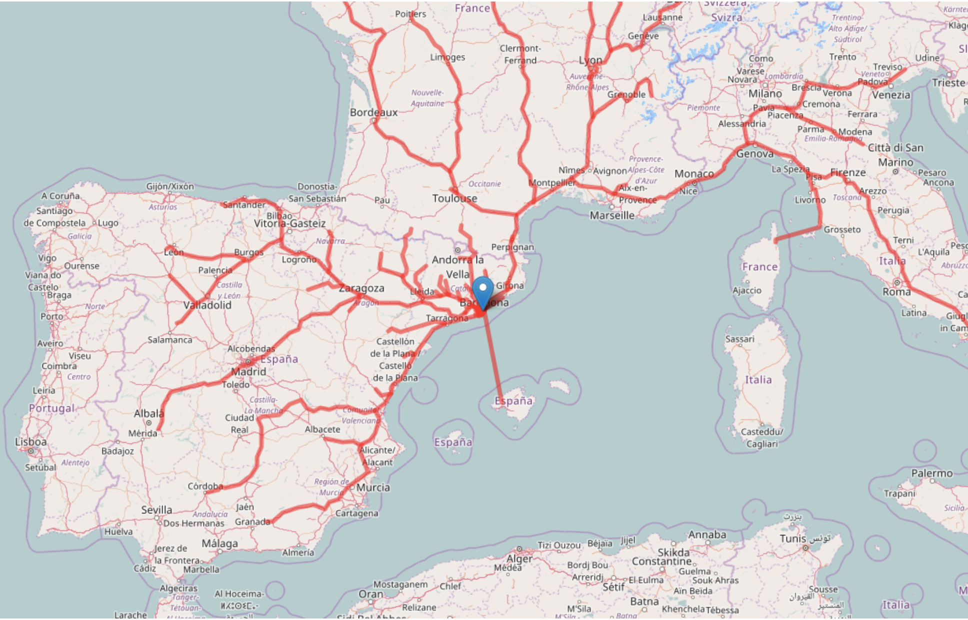

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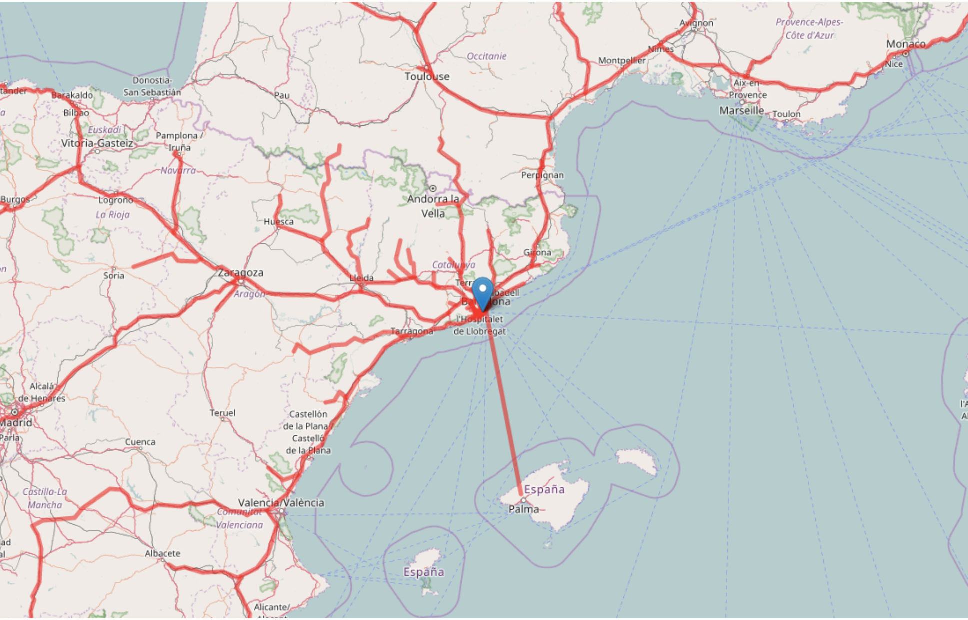

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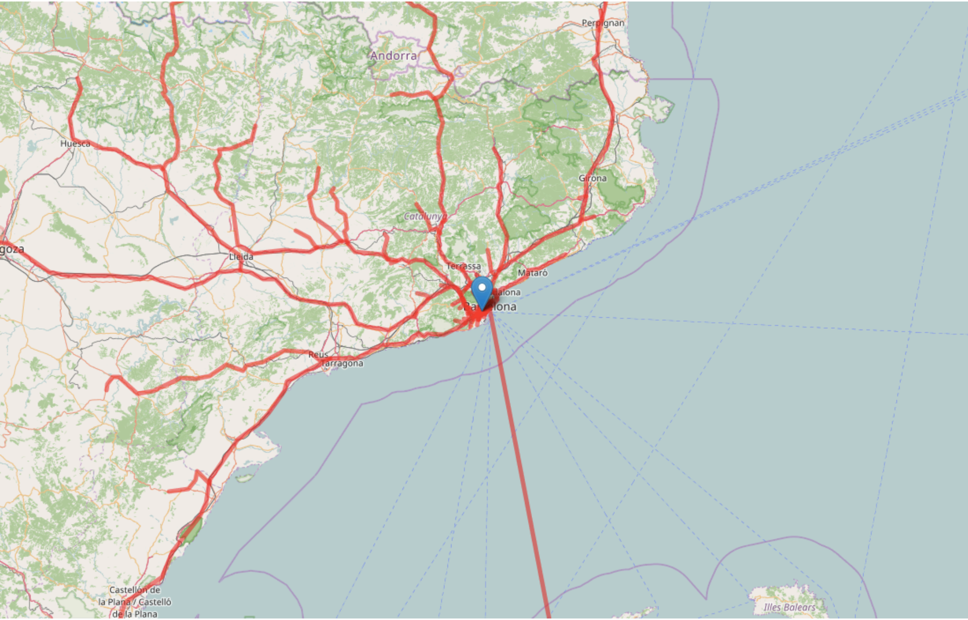

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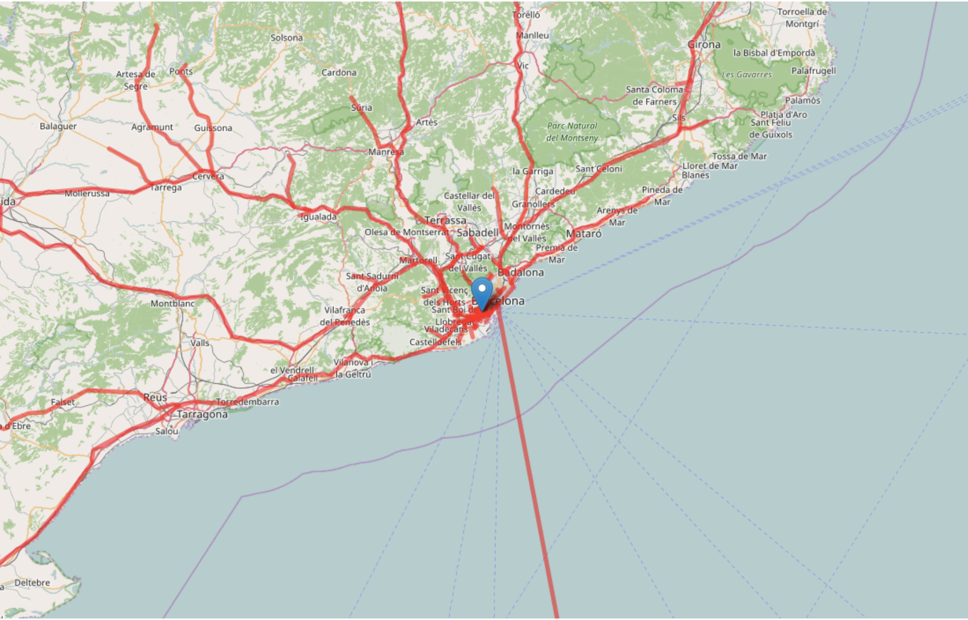

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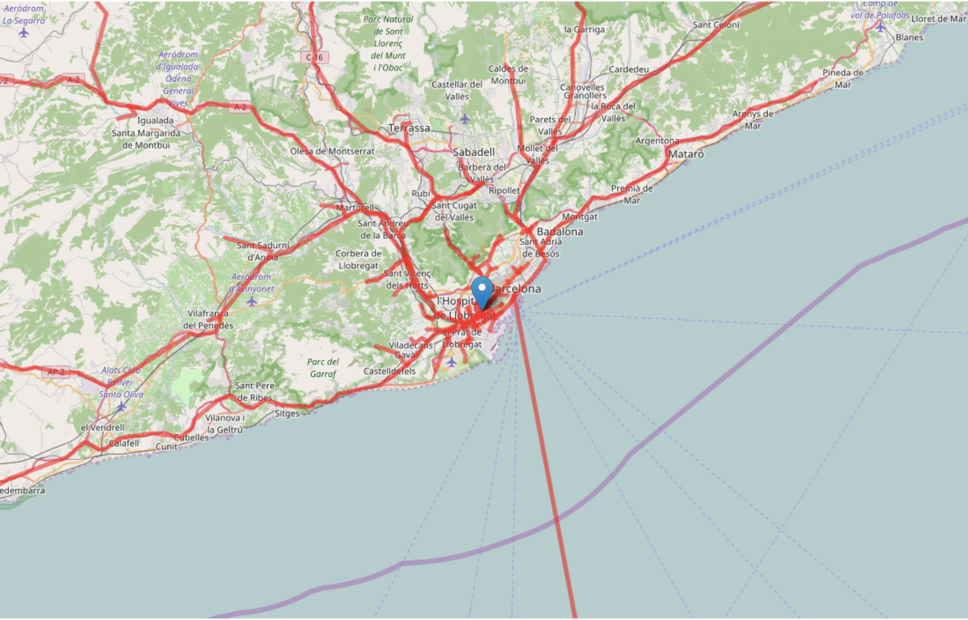

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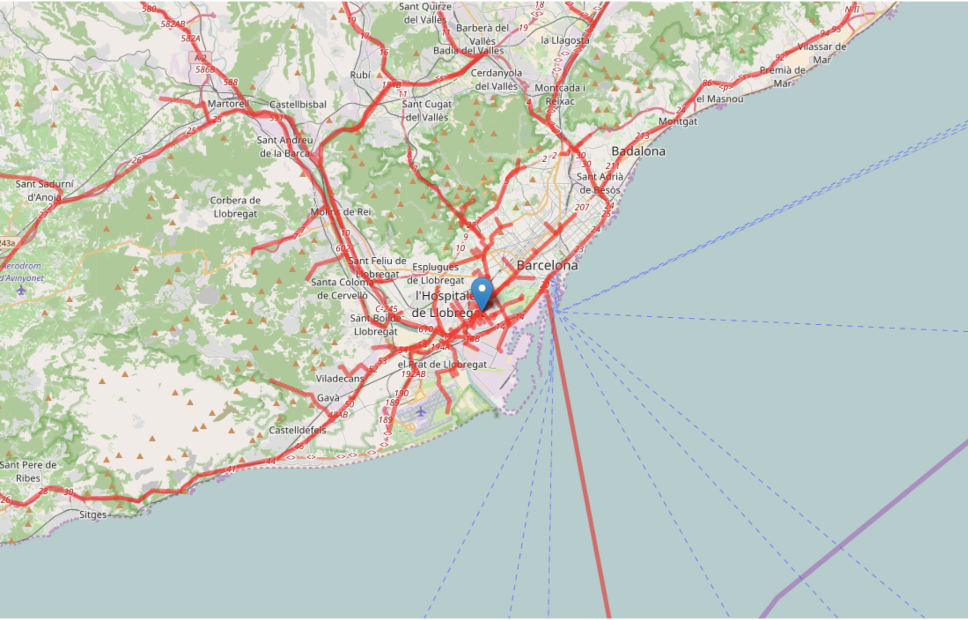

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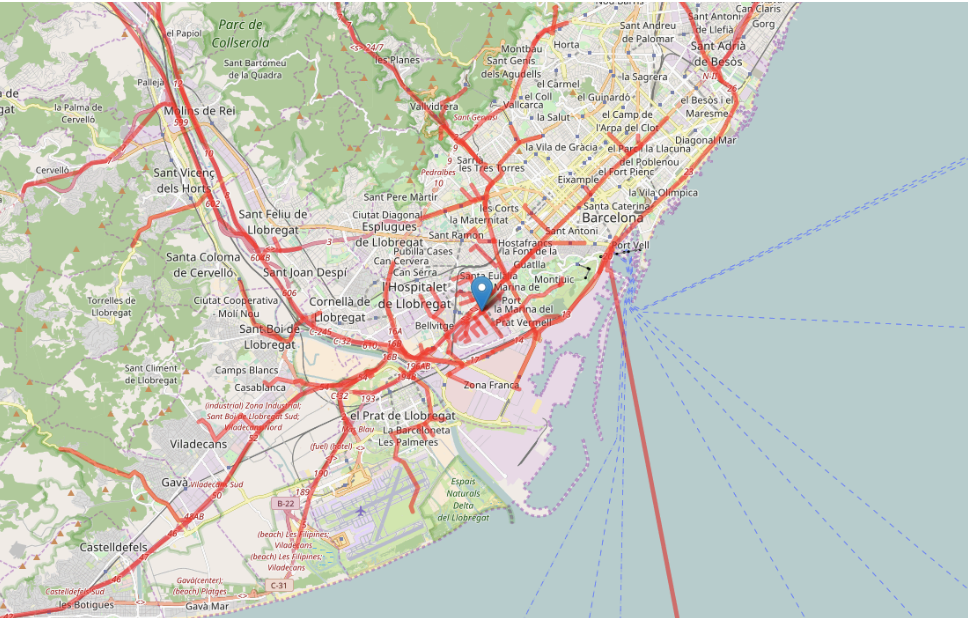

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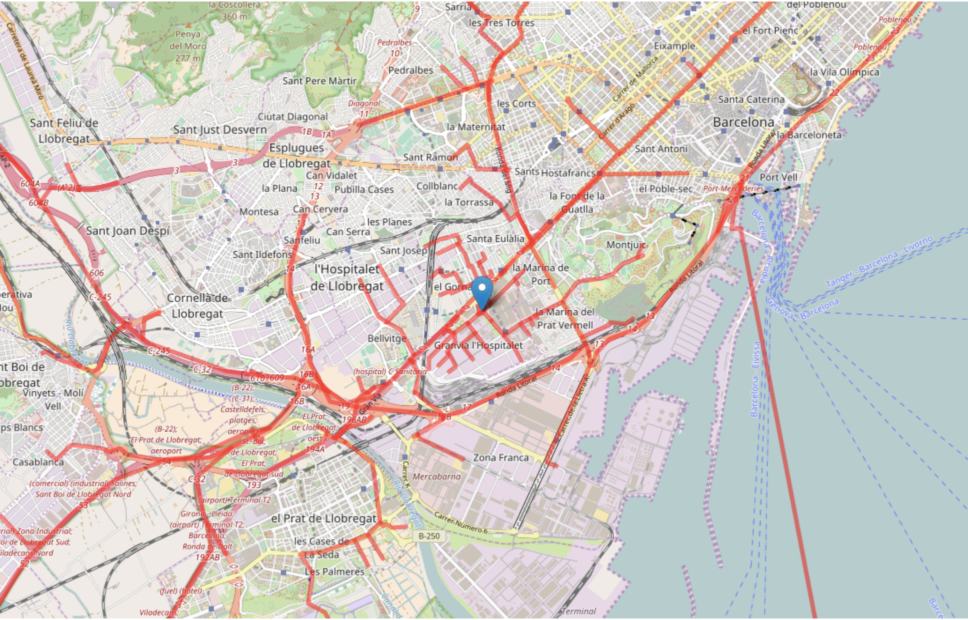

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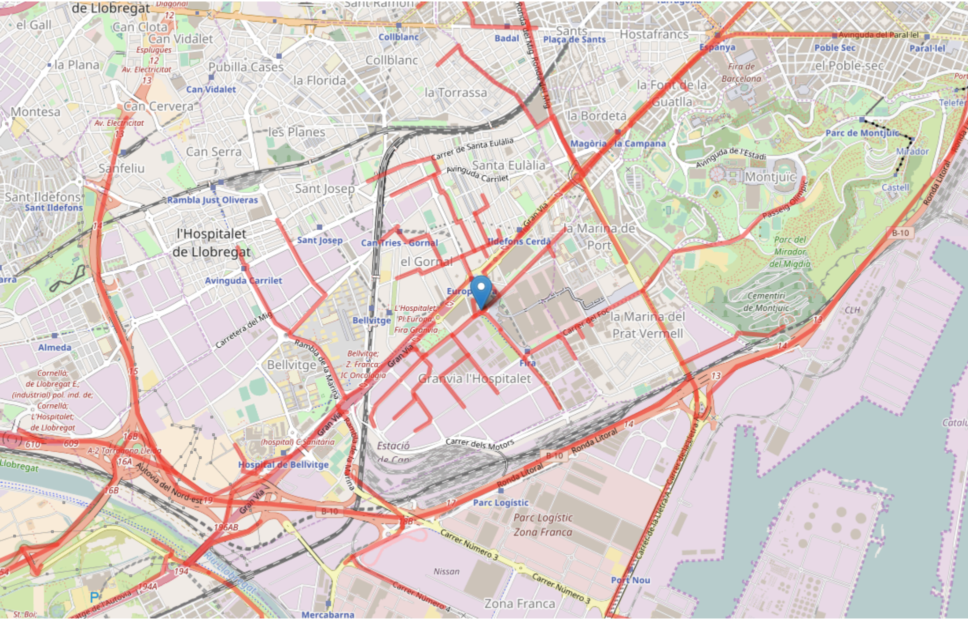
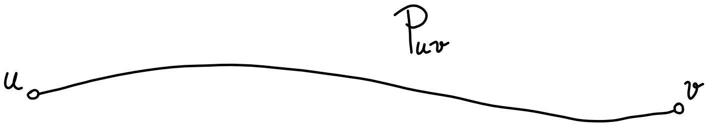


Tree skeleton



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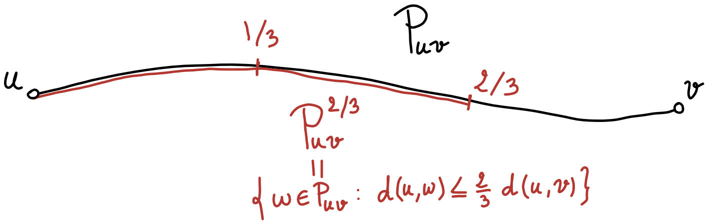

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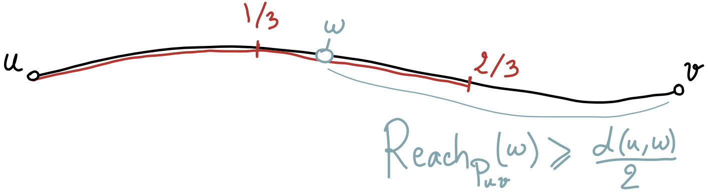

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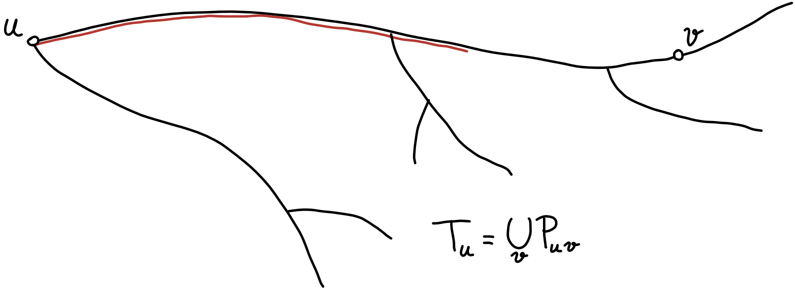

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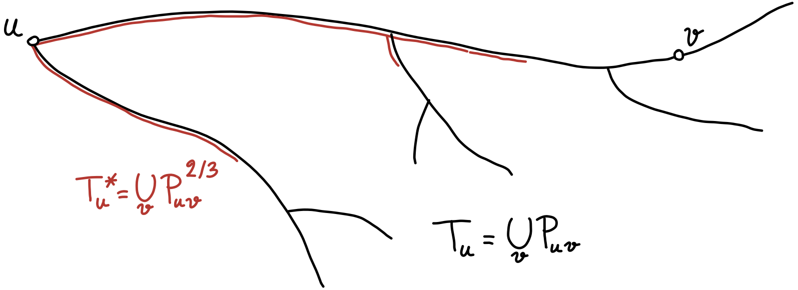

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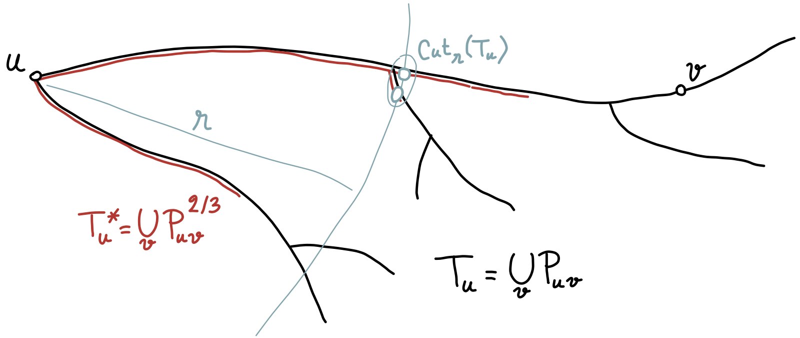

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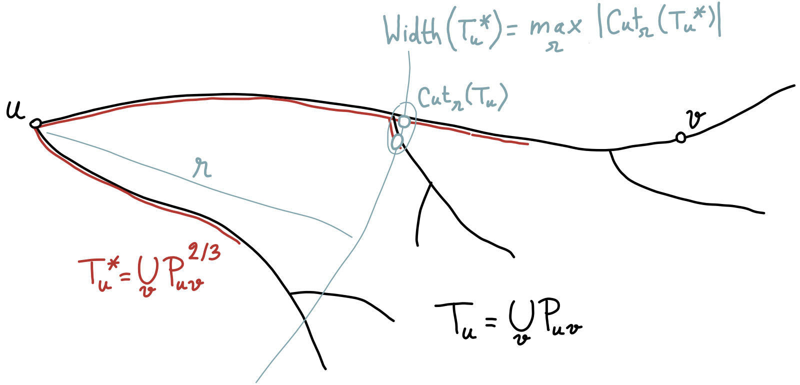

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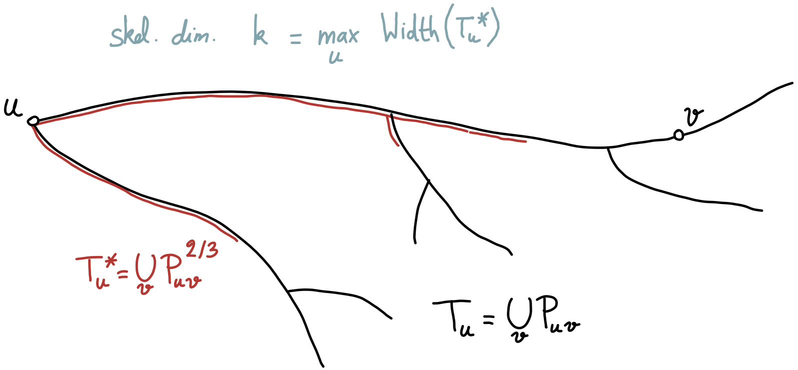
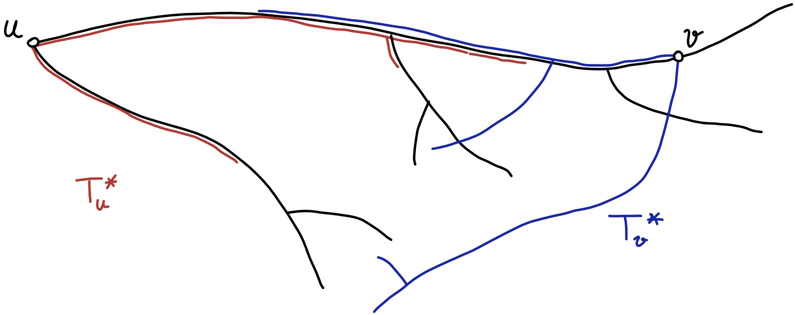


Theorem (Kosowski, V., SODA'17)

Given a graph G with skeleton dimension k and diameter D, a simple random sampling technique allows to find in polynomial time hub sets with size $O(k \log D)$ on average and maximum size $O(k \log \log k \log D)$ with high probability.

Hub set selection : random sampling



The probability to select a node x is $\propto \frac{1}{d(u,x)}$.

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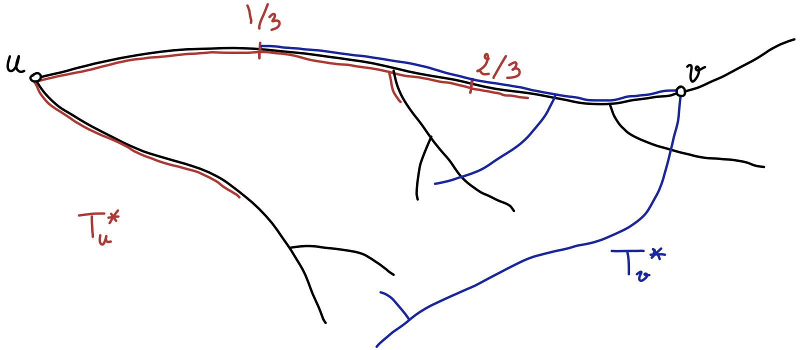

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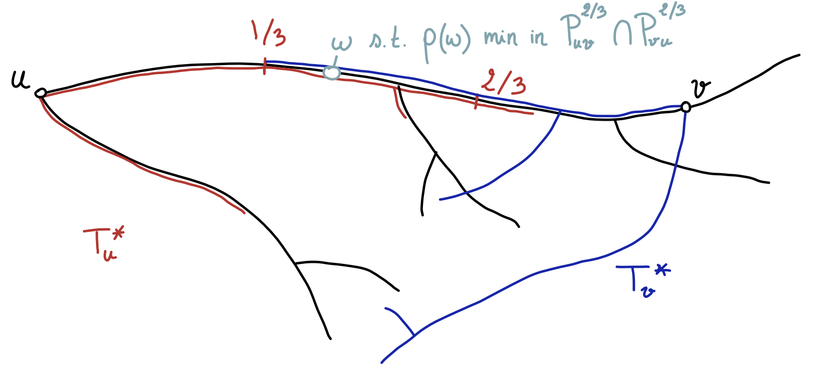

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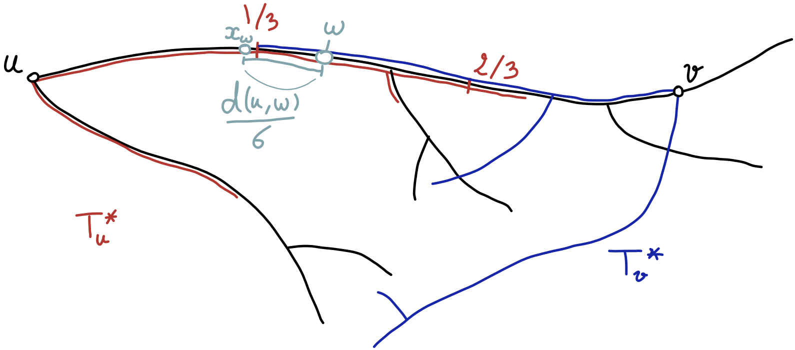

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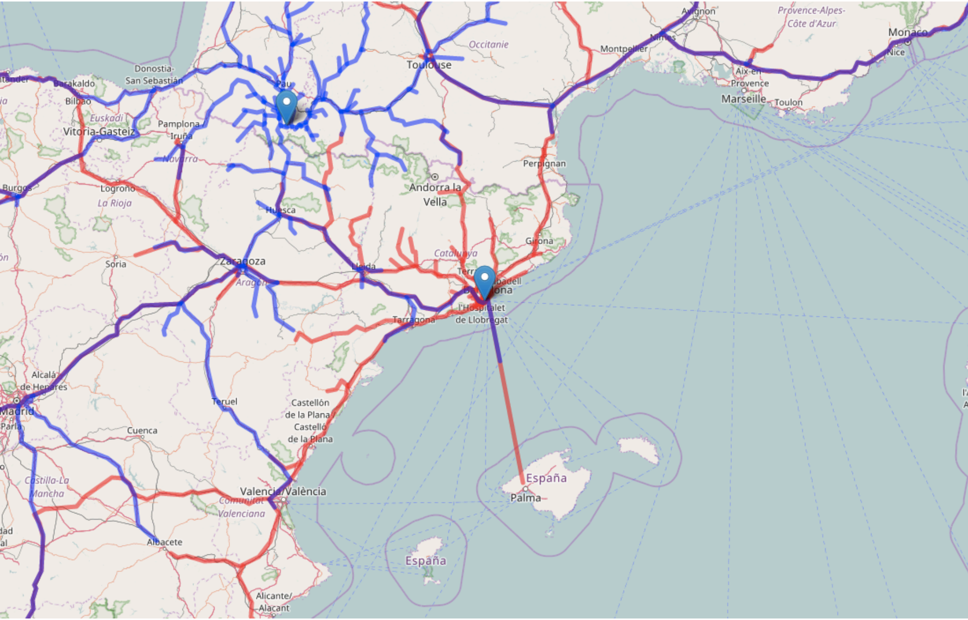
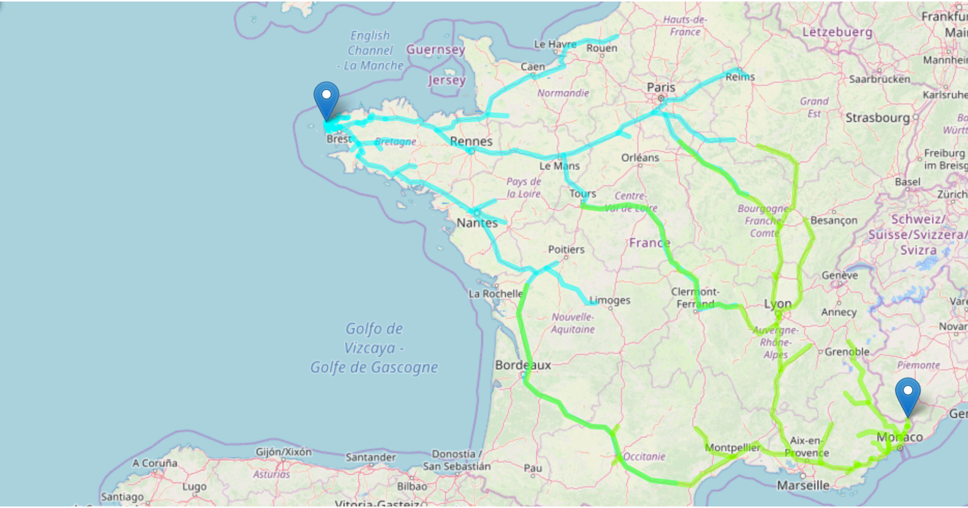
Road networks : two tree skeletons



What ...maps do ?



English
Channel
- La Manche
Guernsey
Jersey
Le Havre
Rouen
Caen
Hauts-de-
France
Lëtzebuerg
Normandie
Paris
Reims
Saarbrücken
Grand
Est
Strasbourg
Brest
Bretagne
Rennes
Le Mans
Orléans
Pays de
la Loire
Tours
Centre-
Val de Loire
Nantes
Bourgogne-
Franche-
Comté
Besançon
Basel
France
Poitiers
La Rochelle
Limoges
Clermont-
Ferrand
Lyon
Genève
Annecy
Nouvelle-
Aquitaine
Auvergne-
Rhône-
Alpes
Grenoble
Golfo de
Vizcaya -
Golfe de Gascogne
Bordeaux
Piemonte
Monaco
Occitanie
Montpellier
Aix-en-
Provence
Marseille
Gijón/Xixón
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Donostia /
San Sebastián
A Coruña
Asturias
Pau
Lugo
Bilbao
Santiago

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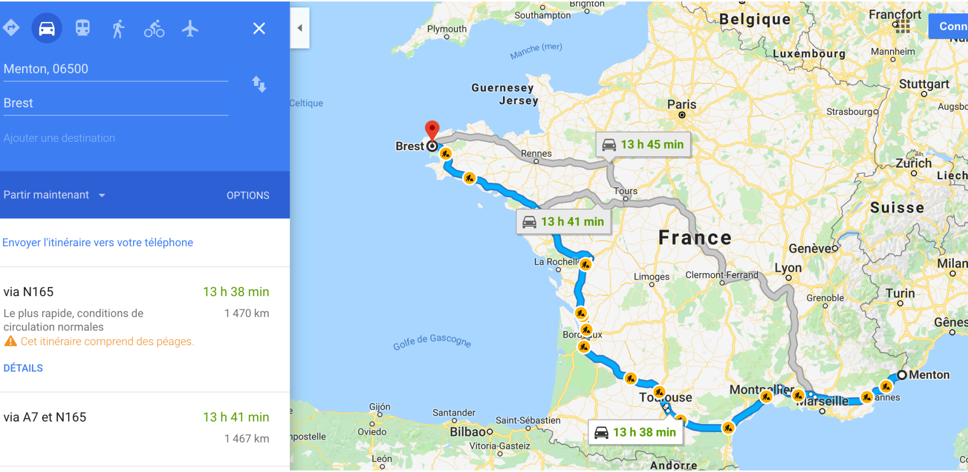

Menton, 06500
Brest
Ajouter une destination
Partir maintenant
OPTIONS
Envoyer l'itinéraire vers votre téléphone
via N165
13 h 38 min
Le plus rapide, conditions de circulation normales
1 470 km
Cet itinéraire comprend des péages.
DÉTAILS
via A7 et N165
13 h 41 min
1 467 km
13 h 45 min
13 h 41 min
13 h 38 min
Brest
Rennes
Paris
Tours
France
La Rochelle
Limoges
Clermont-Ferrand
Lyon
Genève
Grenoble
Turin
Milan
Menton
Marseille
Montpellier
Toulouse
Andorre
Belgique
Luxembourg
Francfort
Mannheim
Stuttgart
Strasbourg
Zurich
Suisse
Guernesey
Jersey
Plymouth
Southampton
Brighton
Manche (mer)
Golfe de Gascogne
Gijón
Oviedo
Santander
Bilbao
Saint-Sébastien
Vitoria-Gasteiz
León

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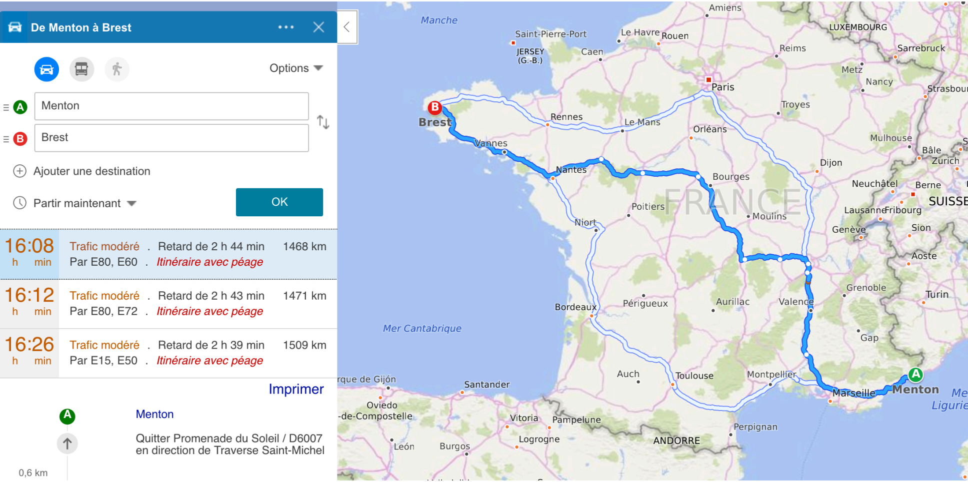
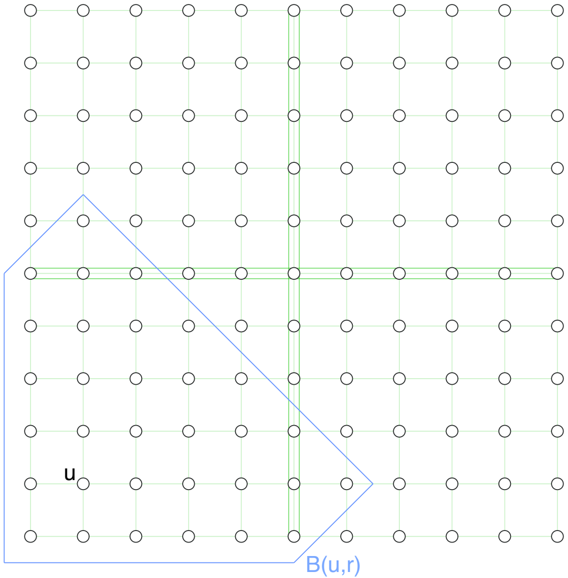

De Menton à Brest
Options
Menton
Brest
Ajouter une destination
Partir maintenant
OK
16:08 h min
Trafic modéré . Retard de 2 h 44 min 1468 km
Par E80, E60 . Itinéraire avec péage
16:12 h min
Trafic modéré . Retard de 2 h 43 min 1471 km
Par E80, E72 . Itinéraire avec péage
16:26 h min
Trafic modéré . Retard de 2 h 39 min 1509 km
Par E15, E50 . Itinéraire avec péage
Imprimer
Menton
Quitter Promenade du Soleil / D6007
en direction de Traverse Saint-Michel
0,6 km
Manche
Amiens
Saint-Pierre-Port
Le Havre
Rouen
LUXEMBOURG
JERSEY (G.-B.)
Caen
Reims
Sarrebruck
Metz
Nancy
Paris
Strasbou
Brest
Rennes
Le Mans
Troyes
Orléans
Mulhouse
Vannes
Dijon
Bâle
Zurich
Nantes
Bourges
Neuchâtel
Berne
FRANCE
SUISSE
Poitiers
Moulins
Lausanne
Fribourg
Niort
Genève
Sion
Aoste
Grenoble
Turin
Bordeaux
Périgueux
Aurillac
Valence
Mer Cantabrique
Gap
Auch
Toulouse
Montpellier
Santander
Oviedo
-de-Compostelle
Marseille
Menton
Vitoria
Pampelune
Logrogne
Perpignan
ANDORRE
León
Burgos

Highway vs skeleton in Brooklyn

Packing of 172 paths

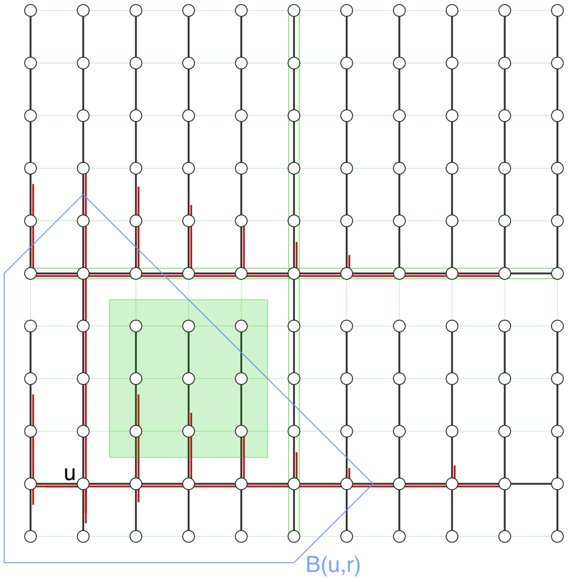
Skeleton width 48

Skeleton dimension of grids



$k = \Theta(\log n)$

Skeleton dimension of grids



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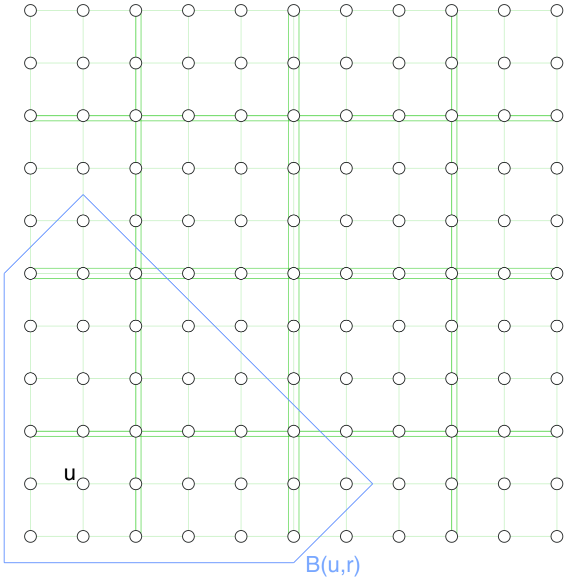

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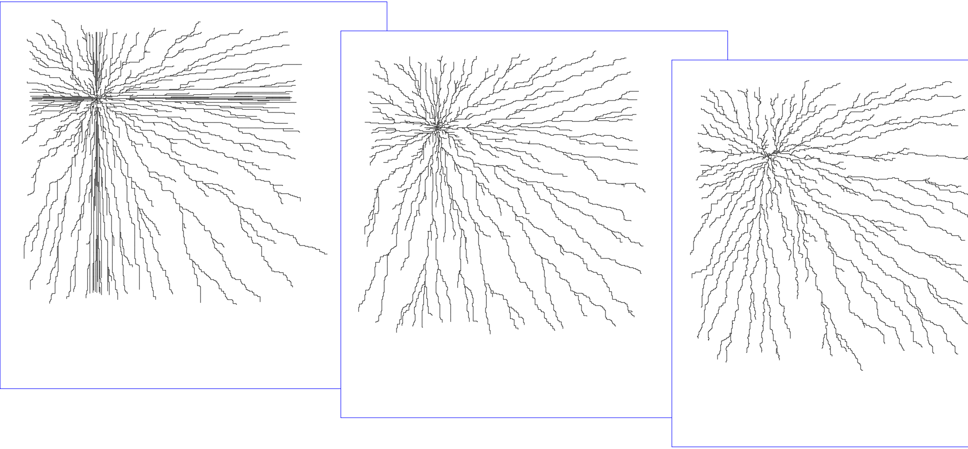
Open : random grid (here 500×500)



k $= 70$ k $= 49$ (fpp $[1, 4)$) k $= 49$ (prob 2/3)

Related to first-passage percolation [Licea, Newman, Piza '96] [Aldous '14].

Part II : Do sparse graphs have covering hub sets with $o(n)$ size?

Can we have sublinear size for sparse graphs ($m = O(n)$)?

Or even constant degree graphs?

Best known upper bound is $O(\frac{n}{\log n})$. [Alstrup, Dahlgaard, Beck, Knudsen, Porah '16] [Gawrychowski, Kosowski, Uznanski '16]

Best known lower bound for distance labels is $\Omega(\sqrt{n})$. [Gavoille, Peleg, Pérennes, Raz '04]

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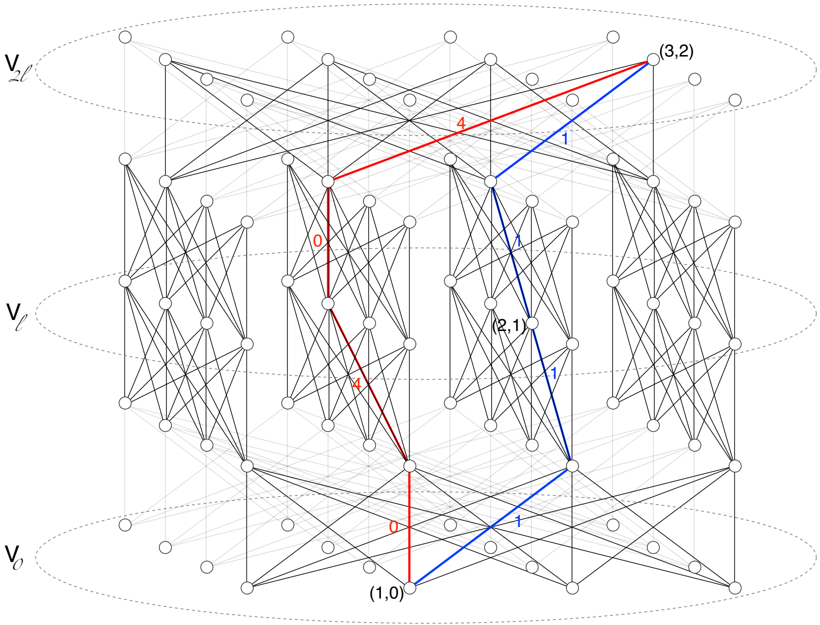
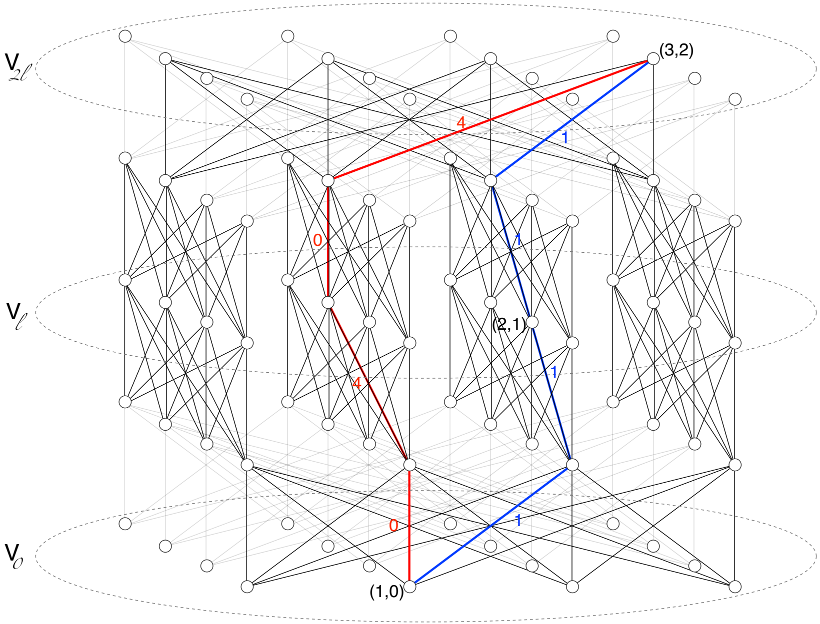
Theorem (Kosowski, Uznański, V., PODC'19)

(1) There exists graphs of degree at most 3 where any collection of covering hub sets has average size $\frac{n}{2^{O(\sqrt{\log n})}}$.

(2) Any graph has a collection of hub sets of $O(\frac{n}{RS(n)^{1/7}})$ size where $2^{\Omega(\log^* n)} \leq RS(n) \leq 2^{O(\sqrt{\log n})}$ is a number related to Ruzsa-Szemerédi graphs.

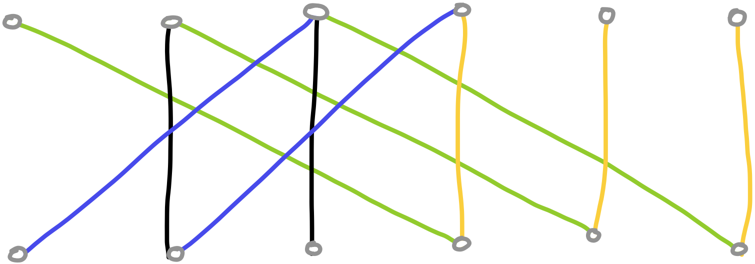
Proof : cov. hub sets of this graph have size $\frac{n}{2^{O(\sqrt{\log n})}}$





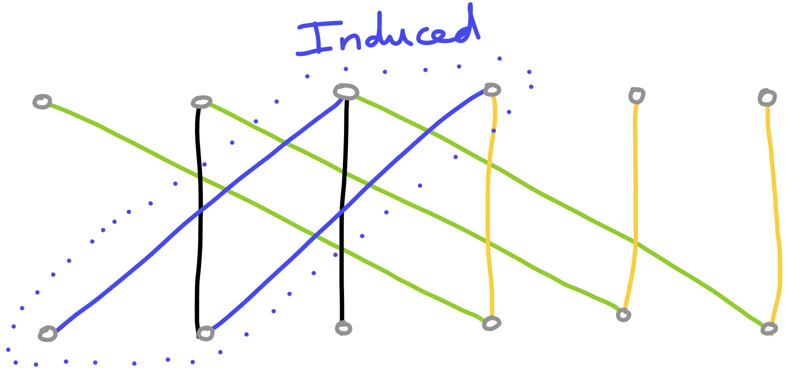
Each V_i is a regular $2^\ell \times \cdots \times 2^\ell$ lattice of dim. $\ell \approx \sqrt{\log n}$ (here $\ell = 2$).
Edges from V_{i-1} to V_i connect nodes differing on ith coordinate.

Ruzsa-Szemerédi



A graph is an RS-graph if it can be decomposed into n induced matchings.

Ruzsa-Szemerédi



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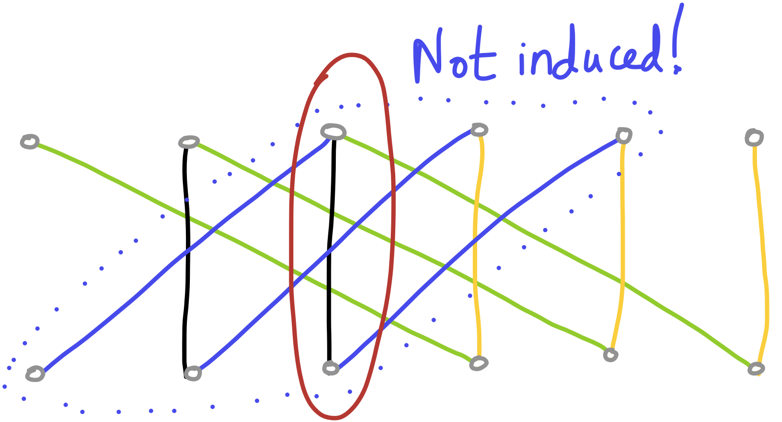

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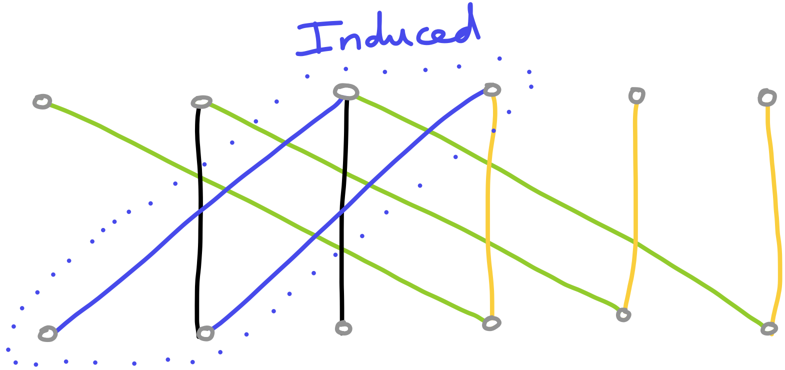

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What are the densest RS-graphs?

Theorem ([Ruzsa, Szemerédi '78])
Any RS-graph has at most $\frac{n^2}{2^{O(\log^* n)}}$ edges.

Define $RS(n)$ as the smallest integer such that there exists an RS-graph with n nodes and $\frac{n^2}{RS(n)}$ edges.

$$2^{\Omega(\log^* n)} \leq RS(n) \leq 2^{O(\sqrt{\log n})}$$

[Ruzsa, Szemerédi '78] [Elkin '10] [Fox '11]

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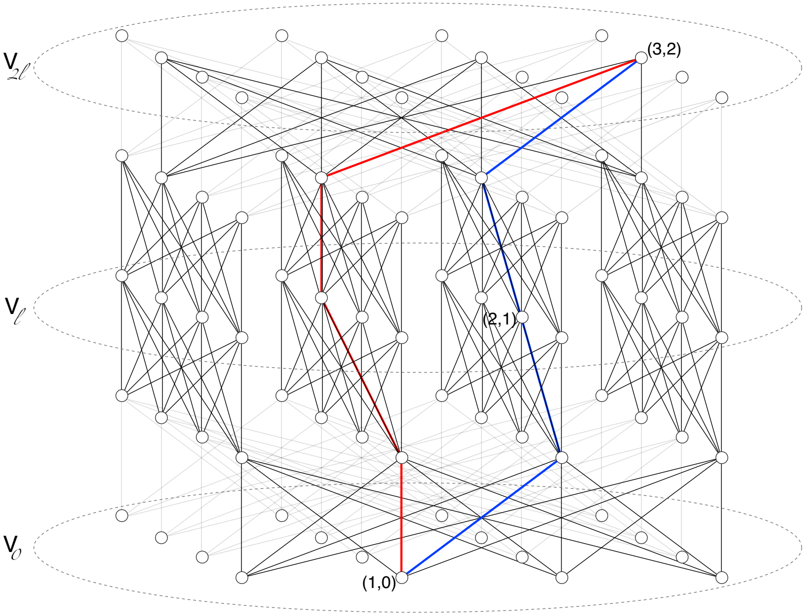


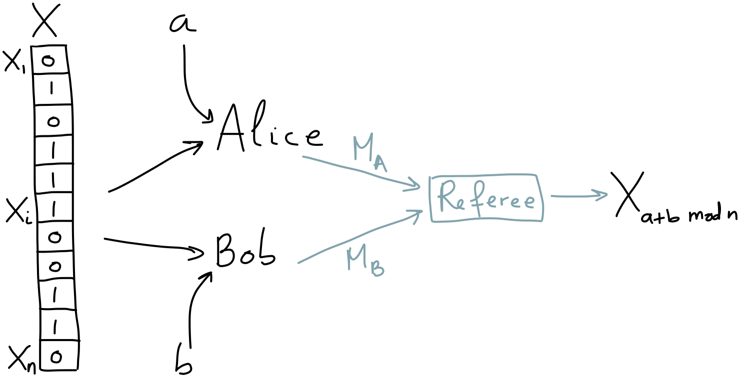
$$G_y^D = \{x_0 z_{2\ell} \mid y = \frac{x+z}{2} \text{ and } d_G(x,z) = D\} \quad \exists D \text{ s.t. } |\cup_y G_y^D| \geq \frac{n^2}{2^{O(\sqrt{\log n})}}$$

Converse

Any cst. deg. graph G has hub sets of av. size $O(\frac{n}{\mathrm{RS}(n)^{1/7}})$.

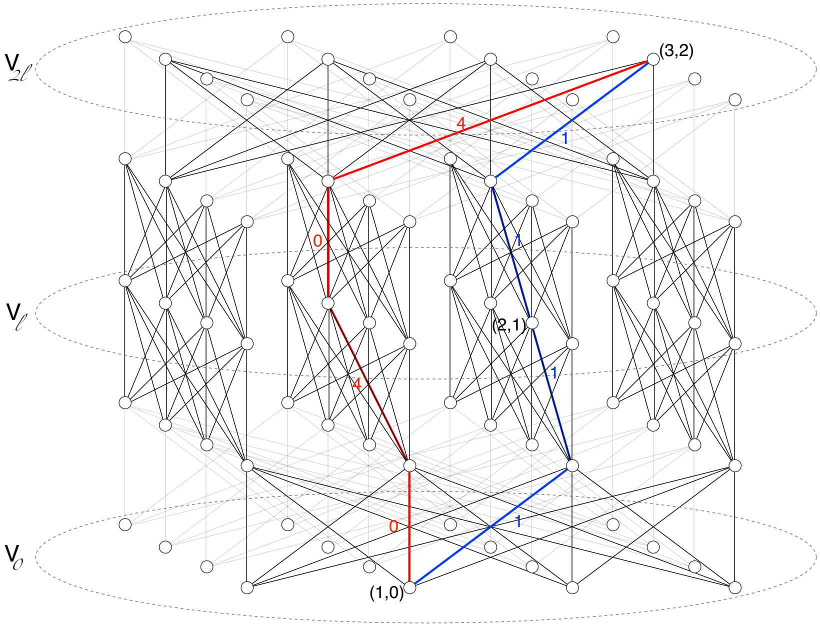
Idea : use a vertex cover of each ${}^{c}G_{y}^{D}$ (VC ≤ 2MM).

Connection with SumIndex problem (comm. complexity)



$\text{SUMINDEX}(n) = \min_{\text{Encoder}} \max_X |M_A| + |M_B|$

$\Omega(\sqrt{n}) \leq \text{SUMINDEX}(n) \leq \widetilde{O}\left(\frac{n}{2^{\sqrt{\log n}}}\right)$ [Pudlak 1994]



$G_X = G \setminus \{y_\ell \mid X_y = 0\}$, send $x = 2a, L_{x_0}$, $z = 2b, L_{z_{2\ell}}$, check $d(x_0, z_{2\ell})$.

Part III : what about 3 hops ?

3-hopset of a path

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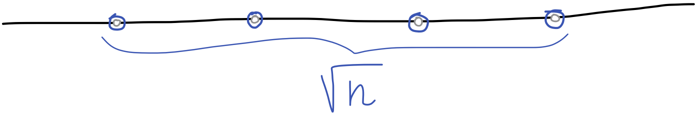
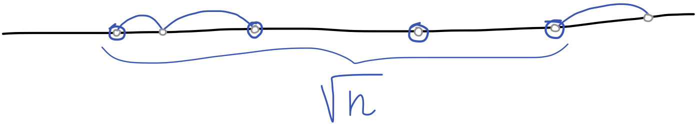


3-hopset of a path



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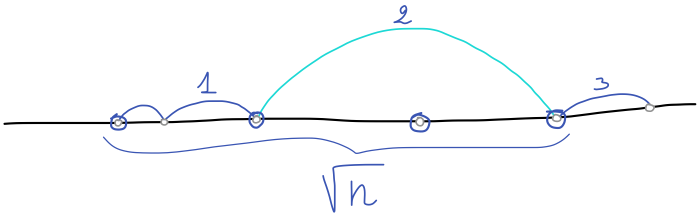

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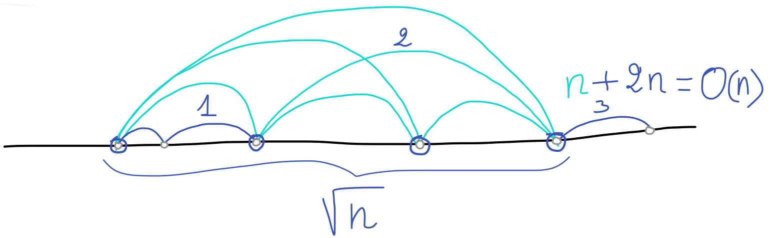

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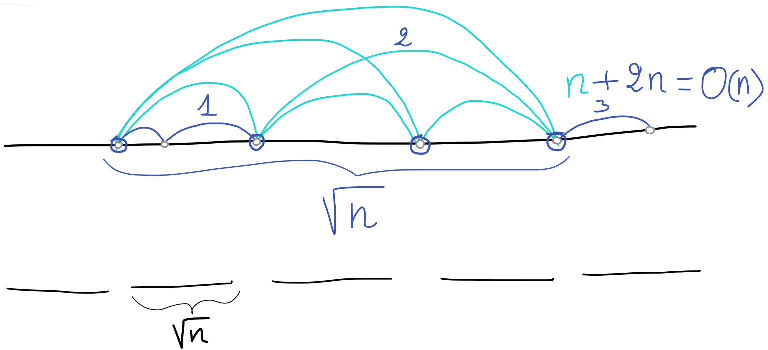

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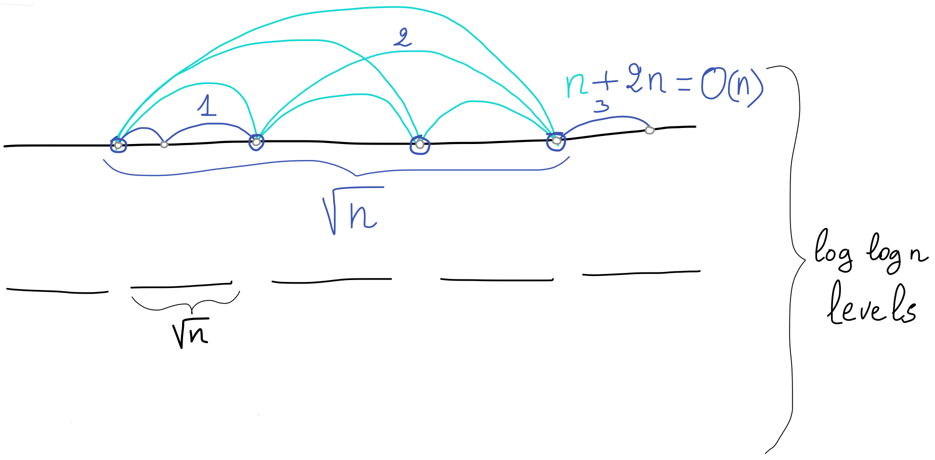

3-hopset distance oracle

Store $x, d_G(u, x)$ for $x \in N_{13}(u)$ ($2 \log \log n$ per node).

Store midle links in a hashtable H_2 ($O(n \log \log n)$ size).

Query for $d_G(u, v)$: best 3-hop path length is

$$\min_{x \in N_{13}(u), y \in N_{13}(v), xy \in H_2} d_G(u, x) + d_G(x, y) + d_G(y, v)$$

($0((\log \log n)^2)$ time).

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Theorem (Kosowski, Gupta, V., ICALP'19)

For a unique-shortest-path graph with skeleton dimension k and average link length $L \geq 1$, there exists a randomized construction of a 3-hopset distance oracle of size $|H| = O(nk \log k(\log\log n + \log L))$, which performs distance queries in expected time $O(k^2 \log^2 k(\log^2 \log n + \log^2 L))$.

Some questions

Improve lower-bounds on sparse graphs for general distance labelings/oracles.

What is the skeleton dimension of a random grid?

What graphs have covering hub sets of size $O(1)$?

What if the graph evolves with time (temporal graphs)?

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Thanks.