Degree-based Outliers Detection within IP Traffic Modelled as a Link Stream

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Abstract—Precise detection and identification of anomalous events in IP traffic are crucial in many applications. This paper intends to address this task by adopting the link stream formalism which properly captures temporal and structural features of the data. Within this framework we focus on finding anomalous behaviours with the degree of IP addresses over time. Due to diversity in IP profiles, this feature is typically distributed heterogeneously, preventing us to find anomalies. To deal with this challenge, we design a method to detect outliers as well as precisely identify their cause in a sequence of similar heterogeneous distributions. We apply it to a MAWI capture of IP traffic and we show that it succeeds at detecting relevant patterns in terms of anomalous network activity.

I. INTRODUCTION

Temporal and structural features of IP traffic are and have been for several years the subject of multiple studies in various fields. A significant part of this research is devoted to detecting statistically anomalous traffic subsets referred to as anomalies, events or outliers. Their detection is particularly important since, in addition to a better understanding of IP traffic characteristics, it could prevent attacks against on-line services, networks and information systems. Methods used in this branch are very broad and depend on both the way in which IP traffic data is modelled and in the statistical analysis used.

Most previous works focus either on the temporal [3], [4] or structural features of traffic [17], [27], with few attempts to combine them [12], [2]. In this paper, we model IP traffic as a link stream which fully captures the both temporal and structural nature of traffic [18], [25]. More specifically, a link stream $L$ is defined as a set of instants $T$, a set of nodes $V$ (IP addresses) and a set of interaction $E$ (communication between IP addresses over time). Within this framework, we focus on one key property: the degree of nodes. We show that this feature is highly heterogeneous, which raises challenges for its use in outlier detection, but it is stable over time. Our method takes advantage of this temporal homogeneity: it divides the link stream into time slices and then performs outlier detection to find time slices which exhibit unusual number of nodes having a degree within specific degree classes. Then, in order to isolate responsible IP addresses and instants on which they behave unexpectedly, we design an identification method based on an iterative removal of previously detected events. Finally, we validate our method by showing that these events removals do not significantly alter the underlying normal traffic.

The paper is organised as follows. First, we overview the related work in Section II. We introduce IP traffic modelling as a link stream and the degree notion in Section III. In Section IV, we describe our goals and the challenges they raise. This leads to the development of our method to detect events in Section V and to identify them in Section VI. Finally, we discuss our results and conclude in Section VII.

II. RELATED WORK

Techniques of anomaly detection in IP traffic are extremely diverse. Among those, methods using principal component analysis [16], [22], machine learning [26], data mining [19], signal analysis and graph-based techniques have been proposed. Concerning signal analysis and graph-based techniques, an important difference lies in data modelling. On one hand, anomaly detection using signal analysis consists in modelling the data as a temporal signal and then spotting anomalies in the Fourier domain at characteristic frequencies. Even if this approach gives powerful results, some structural aspects of the data are lost [3], [4]. The graph approach on the other hand consists in choosing an observation window of a given size and aggregating the links and nodes appearing during this period to form a static graph. The evolution and the structure of exchanges are then observed and studied via a sequence of static graphs obtained either by translation of the observation window or by aggregation on consecutive windows [17], [27]. Thus, it is assumed that all interactions over the same period of time are comparable, which destroys many temporal aspects. Iliofotou et al. [12] use this representation. They introduce several metrics to study similarities between structural features of two consecutive snapshots. They are able to detect changes of behaviours but not specific sub-graphs. Asai et al. [2] use a different graph approach. They include the temporal information directly into the graph: a node is an interaction and two nodes are linked together if they have a causal relationship. In this way, authors manage to detect abnormal temporal and structural sequences. However, this method is limited by the definition of causal relationships that can not take into account all interactions’ properties.

A strength of our work is to preserve both temporal and structural aspects by using the link stream formalism [18], more suited to the data coming from IP traffic. Besides data
modelling, much work has been devoted to the study of anomalies as deviations of the overall traffic volume, like for instance the number of exchanged packets or bytes during a certain amount of time [3], [15]. Link stream formalism allows on the contrary to use more sophisticated features combining both time and structure [18], [25]. The degree for example quantifies the neighbourhood of each node at each moment. Hence, in addition to large events that disrupt traffic volume like flash crowd or alpha flows attacks, it would enable us to detect more subtle and more structurally complex anomalies.

A significant part of our work in this paper is devoted to finding outliers in the degree distribution which is heterogeneous. Up to our knowledge, there is no work dealing exclusively with outliers detection in heterogeneous distributions. Nevertheless, many papers study dissimilarities between different distributions in a way similar to what we do in this paper. Anceaueme et al. [1] and Tajer et al. [24] summarize data streams into sketches and apply a distance metric to quantify the similarity between two sketches. Here again, the data used is aggregated and there is a loss of information. Schieber et al. [23] quantify topological difference between two graphs through Jensen-Shannon divergence and a measure of the heterogeneity of each graphs in terms of connectivity distance between nodes. In both cases, methods used only give a similarity score between two different distributions. Our method, besides quantifying a dissimilarity, gives us additional information: where is the dissimilarity located within the distribution, which then makes it possible to identify its origin. Other studies point to this direction, La Fond et al. [14] propose various measurements allowing the comparison of distributions from one time step to the next, but only recover anomalous time slices. Harshaw et al. [10] achieve to find anomalous IP addresses and time slices in series of graphs by comparing, for each graph, the count of specific sub-graphs describing their topology. However, they are still subject to information loss coming from data modelling.

III. TRAFFIC MODELLING AS A LINK STREAM AND DEGREE DEFINITION

IP traffic consists of packet exchanges between IP addresses. We use here one hour of IP traffic capture from the MAWI archive\(^1\) on June 25th, 2013, from 00:00 to 01:00. We denote this trace by a set \(D\) of triplets such that \((t, u, v) \in D\) indicates that IP addresses \(u\) and \(v\) exchanged at least a packet at time \(t\). The set \(D\) contains 83,386,538 triplets involving 1,157,540 different IP addresses.

We model this traffic as a link stream \(L\) in order to capture its structure and dynamics [18]. In this link stream, nodes are IP addresses involved in \(D\) and two nodes are linked together from time \(t_1\) to time \(t_2\) if they exchanged at least one packet every second within this time interval. Formally, \(L = (T, V, E)\) is defined by a time interval \(T \subseteq \mathbb{R}\), a set of nodes \(V\) and a set of links \(E \subseteq T \times V \times V\) where \(V \times V\) denotes the set of unordered pairs of distinct elements of \(V\), denoted by \(uv\) for any \(u\) and \(v\) in \(V\) (thus, \(uv \in V \times V\) if and only if \(u, v \in V\) and \(u \neq v\), and we make no distinction between \(uv\) and \(vu\)). If \((t, u, v) \in E\) then \(u\) and \(v\) are linked together at time \(t\). In our case, \(E = \cup_{(t, u, v) \in D} [t - \frac{1}{2}, t + \frac{1}{2}] \times \{uv\}\). See Figure 1.a for an illustration.

In \(L\), the degree of \((t, v) \in T \times V\), denoted by \(d_t(v)\), is the number of distinct nodes with which \(v\) interacts at time \(t\):

\[d_t(v) = |\{u, (t, uv) \in E\}|.\]

Figure 1.b shows the degree of node \(b\) over time.

IV. HETEROGENEITY OF DEGREES

In order to find events in a link stream using the degree, we first need to characterize the normal behaviour of couples \((t, v)\) with respect to this feature. Then, a couple \((t, v) \in T \times V\) having a significantly different degree from the one of others would indicate an event: \(v\) interacts with an unusually high number of nodes at time \(t\).

For this purpose, we call degree distribution of \(L\) the fraction \(f(k)\) of couples \((t, v) \in T \times V\) for which \(d_t(v) = k\), for all \(k\):

\[f(k) = \frac{|\{t, v) \in T \times V : d_t(v) = k\}|}{|T \times V|}.\]

Figure 2 shows that the degree distribution is very heterogeneous, which discards the hypothesis of a normal

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\(^1\)http://mawi.wide.ad.jp/mawi/ditl/ditl2013/ [13]
behaviour. In this situation, one may hardly identify values of degree that could be considered anomalous. Indeed, in such heterogeneous distributions, the mean and the standard deviation, which characterize the statistical normal behaviour, are not good estimators.

In order to circumvent this global heterogeneity, we observe degrees on sub-streams corresponding to IP traffic during time slices of two seconds. Formally, we call \( T_i = \{2i, 2i+2\} \) the \( i^{th} \) time slice, for all \( i \in \{0, \ldots, 1799\} \) such that \( T_0 = [0, 2] \) and \( T_{1799} = [3598, 3600] \), and we define

\[
f_i(k) = \frac{\left| \{(t, v) \in T_i \times V : d_i(v) = k\} \right|}{|T_i \times V|},
\]

the degree distribution of the \( i^{th} \) time slice. Figure 3 shows that these distributions also are heterogeneous.

However, Figure 3 also shows that degree distributions \( f_i \) have similar shapes. To quantify this similarity, we fit them with a power law model, \( y \propto x^a \), where we estimate the power law exponents by using linear fits of the distributions in which both coordinates are log-transformed. Other more complex and accurate techniques to fit power laws exist, see for instance [7]. Note that, in our context, the goodness of the fit is not the outcome of greatest interest. We are interested in knowing whether the parameters are similar on all time slices or not, not in values taken by parameters. We see on Figure 4 that linear model parameters are homogeneously distributed, suggesting that even if nodes have behaviours that are not comparable with each other, their overall behaviour is comparable from one time slice to another. We also see outlier values, distant from the mean, which in turn indicate changes in the overall behaviour on particular sub-streams. Using these observations, we design an outlier detection method based on temporal homogeneity of heterogeneous degree distributions.

**V. LEVERAGING TEMPORAL HOMOGENEITY TO DETECT EVENTS**

The above observations lead to the following conclusion: degree distributions are heterogeneous in the same way on most, if not all, time slices. In other words, in each time slice, the fraction of couples \((t, v)\) that have a given degree is similar to this fraction in other time slices. This is what we will consider as normal. Anomalies, instead, correspond to significant deviation from the usual fraction of nodes having a given degree. In this section we describe our method to compare degree distributions on all time slices and its use for outlier detection in our dataset.

First, notice that it makes little sense to consider the fraction of couples \((t, v)\) having a degree exactly \( k \) when \( k \) is large: having degree \( k-1 \) or \( k+1 \) makes no significant difference. Therefore, we define degree classes \( C_j \) and consider the fraction of couples \((t, v)\) having degrees in \( C_j \), for all \( j \):

\[
f_i(C_j) = \frac{\left| \{(t, v) \in T_i \times V : d_i(v) \in C_j\} \right|}{|T_i \times V|}.
\]
Many options regarding the definition of \( C_j \) may make sense. It seems crucial, however, to distinguish between degree 1 and degree 2, as well as to take into account the heterogeneity of degrees. Therefore, we choose to group them into classes of logarithmic scale. We define here the heterogeneity of degrees. Therefore, we choose to group degree 1 and degree 2, as well as to take into account the sense. It seems crucial, however, to distinguish between

\[
\frac{C_j}{d_j} \text{ distributions for classes } C_j = \{ k_j \}, \ldots, [k_j+1] - 1 \text{ such that } k_1 = 1 \text{ and } \log(k_{j+1}) = \log(k_j) + 0.1, \text{ which leads to } C_1 = \{1\}, C_2 = \{2\}, C_3 = \{3\}, C_4 = \{4, 5\}, \text{ etc., until } C_{41} = \{19953, \ldots, 25117\}. \text{ We leave the exploration of other class constructions for future works.}
\]

In order to compare degree distributions, we plot for a given degree class \( C \), the distribution on all time slices \( T_i \) of the fraction \( f_i(C) \). In other words, we study how the fraction of couples \((t, v)\) having degrees within \( C \) during \( T_i \) is distributed among all time slices. Figure 5 shows the distributions for classes \( C_1, C_2, C_19, C_{22}, C_{31} \) and \( C_{41} \).

In accordance with temporal homogeneity, we can see that most fractions are distributed around the mean and that a few only are distant from it. As expected according to the heterogeneity of degrees, the higher the degree class, the lower the fraction of couples \((t, v)\) within the class. We see on \( C_1 \) that the average fraction over all time intervals is \( 2.1 \cdot 10^{-3} \). When switching to \( C_2 \), it drops to \( 1.15 \cdot 10^{-4} \) and gradually decreases to reach 0 in classes of degrees above 252. In these high degree classes, there is a peak on fraction 0, indicating that in most time slices the normality is that there is no couple \((t, v)\) which have a degree reaching these classes.

In order to validate fractions \( f_i \) homogeneity over time slices within each degree class, we fit their distributions with a normal distribution model \( P(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \) where values are normally distributed around a mean \( \mu \) with a standard deviation \( \sigma \). Deciding whether a given distribution is homogeneous with outliers or not may then be done as follows [17]: (1) iteratively remove outliers from the distribution with Grubbs’ test [8]; (2) fitting the resulting distribution with the normal model; (3) evaluate the goodness of the fit. We use Maximum Likelihood Estimation (MLE) to determine which model parameters fit the best the empirical distribution [6] and evaluate the goodness of the fit with the Kolmogorov-Smirnov (KS) distance between the empirical
and the reference distributions [21]. In this framework, we find 37 distributions homogeneous with outliers among the 41 corresponding to each degree class (see Fig. 6). The remaining 4 are discarded from the study. One may use more complex and accurate techniques to automatically perform this decision, see for instance the work performed by Motulsky et al. [20].

Unlike heterogeneous distributions, homogeneous distributions with outliers clearly exhibit statistical anomalies: most values are similar to a mean value (normality) but some significantly deviate from it (abnormality). Given an homogeneous distribution with outliers, we use here the classical assumption that a value is anomalous if its distance from the distribution is the following: we group degree values into homogeneous distribution with outliers clearly exhibit statistical anomalies: most values are similar to a mean value (normality) but some significantly deviate from it (abnormality). Given an homogeneous distribution with outliers, we use here the classical assumption that a value is anomalous if its distance from the distribution is the following: we group degree values into homogeneous distribution with outliers clearly exhibit statistical anomalies: most values are similar to a mean value (normality) but some significantly deviate from it (abnormality). Given an homogeneous distribution with outliers, we use here the classical assumption that a value is anomalous if its distance from the distribution is the following: we group degree values into

Finally, our method for event detection from degrees distribution is the following: we group degree values into degree classes of logarithmic width. For a given degree class \( C \), we look at the distribution on all time slices of the fraction \( f_i(C) \). This distribution indicates anomalous values which mean that there are anomalous high numbers of couples \((t, v)\) having degree within \( C \) during specific time slices. We then call an anomalous value of this kind a detected event.

A detected event gives two pieces of information: the time slice \( T_i \) on which the anomalous value has been observed and the degree class \( C \) in which the couple(s) responsible for this anomaly are located. At this stage, we detected 1,358 such events. We now address the goal of identifying the couples \((t, v)\) in \( T \times V \) responsible for these detected events.

VI. ITERATIVE REMOVAL TO IDENTIFY EVENTS

A detected event is a degree class \( C \) and a time slice \( T_i \) such that the fraction \( f_i(C) \) is unusually high compared to the ones in other time slices. Identifying this event means recovering the set of couples \((t, v)\) responsible for this anomaly. In this section, we introduce an iterative removal method and show that it leads to such identification.

Let’s take time slice \( T_{1080} \) detected in degree class \( C_2 \) as an example. We have access to the set of couples \((t, v)\) which have a degree in \( C_2 \) during \( T_{1080} \). However, we cannot directly identify the event by this set. Indeed, let’s consider the new link stream \( L' = (T, V, E') \) with \( E' = E \setminus \{(t, v) : t \in T_{1080} \text{ and } d_i(v) \in C_2 \} \). We see on Figure 8 that the removal of this set of interactions from the link stream causes the appearance of a negative outlier\(^2\) in the distribution of the fractions on \( C_2 \). Thus, by removing all interactions \((t, uv)\) such that couples \((t, v)\) have degree in \( C_2 \) during \( T_{1080} \), we removed anomalous traffic but also normal traffic. Therefore, identifying the detected event as the set \( \{(t, v) : t \in T_{1080} \text{ and } d_i(v) \in C_2 \} \) is not accurate enough.

This suggest that one cannot directly identify couples acting abnormally in low degree classes. Indeed, in these classes, the normal fraction is greater than zero. Hence, an anomalous fraction consists in anomalous couples but also normal ones, which prevents us from identifying responsible couples only. On the contrary, in high degree classes the expected fraction is zero. Thus, couples \((t, v)\) contributing to non-zero fractions are clearly anomalous. Events detected in such degree class \( C \) can therefore be correctly identified with the set \( \{(t, v) : t \in T_i \text{ and } d_i(v) \in C \} \).

\(^2\)We call negative outlier an outlier which is lower than the mean.
Consequently, we now consider degree class \( C_{41} \) on which the normal fraction is 0. Its larger anomalous fraction corresponds to time slice \( T_{315} \). Hence, this event can be identified by the set \( \{ (t, v) : t \in T_{315} \text{ and } d_t(v) \in C_{41} \} \). Figure 9 shows the consequences of its removal. As expected, the anomalous fraction in \( C_{41} \) vanishes without creating a negative outlier. Additionally, one may notice the disappearance of an outlier in \( C_1 \). By looking into the data, we can see that the removal set corresponds to a single node, \( u \), whose neighbours all have degree 1. Thus, the outlier that disappears in \( C_1 \) was, in fact, caused by the high number of neighbours of \( u \). The removal of \( u \) and the one of its interactions then lead to the identification of the event in \( C_1 \) by the set \( \{ (t, v) : t \in T_{315} \text{ and } v \in N_t(u) \} \).

Finally, our approach for event identification consists in removing one by one correctly identified events in high degree classes. We repeat this operation until we reach classes of degree in which outliers contain anomalous traffic as well as normal traffic. This iterative process, in addition to removing anomalous traffic identified in high degree classes, allows to identify related events in lower classes as well. If a given removal creates a negative outlier in a degree class, this means that we removed too much. The removal that caused it is then cancelled and the corresponding event stays detected but unidentified.

In our dataset, none of the removals generated negative outliers. Altogether, we directly identified and removed 205 events in high degree classes. These removals allowed us to identify a total of 1,163 outliers on the 1,358 previously detected ones, hence more than 85% of detected outliers. We can see in Figure 10 the final shape of classes \( C_1 \) and \( C_2 \) in which almost all outliers disappeared. Figure 11 shows the degree profiles of 4 nodes which have been removed for time periods during which they were acting abnormally. In particular, node \( v_1 \), for which the degree fluctuates within \( C_{22} \), contributes identically and maximally to the fractions \( x \) of couples \( (t, v) \) within this class during the corresponding interval. This makes it responsible for the peak on fraction \( 8.5 \cdot 10^{-7} \) observed in \( C_{22} \) on Figure 5. As expected, we notice the disappearance of this outlier after the removal of \( v_1 \).

**Fig. 9: Event identification in high degree classes** - The removal of an identified event in the high degree class \( C_{41} \) allows the identification of an event detected in the lower degree class \( C_1 \).

**Fig. 10: Distributions of the fractions on all time slices for degree classes \( C_1 \) and \( C_2 \) after events removals** - Before events removals there were 151 anomalous values in \( C_1 \) and 5 in \( C_2 \). After the removals, it only remains 10 unidentified anomalous values in \( C_1 \) and 2 in \( C_2 \).

**Fig. 11: Degree profiles of 4 identified nodes** - \( v_1 \) is responsible for the high probability on the largest fraction on \( C_{22} \). The set \( \{ (t, v_1) : t \in [712, 940] \text{ and } d_t(v) \in C_{22} \} \) has been identified and removed. \( v_2 \) has a normal activity with a degree around 160 and a sharp variation on \( T_{223} = [446, 448] \). The set \( \{ (t, v_2) : t \in T_{223} \text{ and } d_t(v) \in C_{32} \} \) has been identified and removed. However, normal interactions of \( v_2 \) during this time interval were also removed. The degree of \( v_3 \) reaches several powers of two which indicates that this node is running network scans [11]. The sets \( \{ (t, v_3) \} \) where \( v_3 \) is active have all been identified and removed. For the node \( v_4 \), the four peaks corresponding to degree values higher than 300 have been identified and removed.
we were then able to identify couples \((t, v)\) responsible for this anomaly but only in high degree classes. Hence, we then introduced an event identification procedure relying on an iterative removal of events identified in these high degree classes. This last steps allowed us to trace back responsible IP addresses and instants in low degree classes as well.

The results we obtained show that our method allows to find interesting anomalous activities in IP traffic. In particular, we found point to multipoint anomalies and network scans as for instance node \(v_3\) in Figure 11. More generally, our method succeeds in finding anomalous couples \((t, v)\) independently of their degree’s order of magnitude. Hence, a node having a constant degree will not be identified as anomalous on any time slice even if its degree is much larger than other nodes. It will however detect couples \((t, v)\) acting abnormally compared to what others do on other time slices. These results could not have been obtained by studying degree variations for all couples \((t, v) \in T \times V\). Indeed, studying this feature leads to the same heterogeneity problem: there are nodes that suddenly interact with twice more neighbours, as well as 5 or 100, etc., times more. Hence, we are still confronted with different orders of magnitude and consequently to heterogeneous distributions.

Figure 12 shows the number of packets per second and the number of distinct nodes per second before and after applying our method. These two features are distributed homogeneously with outliers on all seconds within \(T\). However an outlier only tells us that there are seconds during which the number of packets, or the number of distinct nodes, respectively, is larger than usual. Hence, the event is detected but not identified since we cannot trace back responsible nodes nor instants with these distributions only. After removing the events identified with our method, we see that peaks as well as sudden changes in the trends disappear without altering the underlying normal traffic. This means that our method enables us to identify events in other measurements where anomalies had been detected but not identified. In particular for the number of distinct nodes per second, for each outliers in the distribution we know which couples \((t, v)\) caused it. This last result is particularly promising: it shows that by using more complex metrics, it is possible to identify events previously detected but unidentified with simpler metrics. The 195 events that we have not been able to identify with

\footnote{Nodes cannot be identified by newly appeared nodes of degree 1 after the detection of an anomaly. Indeed, the number of new nodes of degree 1 appearing in each sub-stream is much larger than the number of nodes causing the anomaly.}
the degree could therefore be identified in future works by using other features of the link streams.

This work however only is a first step towards anomaly detection in link streams and may be improved on several aspects. In particular, some removals delete anomalous traffic as well as normal traffic without creating a negative outlier. This is the case, for instance, when a node $u$ has a normal activity with a non-zero degree and a sudden change on a time slice as for node $v_2$ in Figure 11. Degree allows to detect couples $(t, u)$ but not specific links $(t, uv)$. Hence, by removing the set of identified couples $(t, u)$ during the detected time slice, we remove $u$’s anomalous interactions as well as $u$’s normal interactions. This last point could be improved by considering more complex features than the degree, defined on the set of interactions $E$ instead of the set of couples $T \times V$. Many details may also be improved, especially the choice of parameters and modelling hypothesis, like for instance: the fact that we linked nodes together if they exchanged packets at least every second; the fact that we considered undirected links; or the effects of classes sizes on the results. One may also explore other choices for the duration of time intervals on which we compare degree distributions.

The next logical step of this work would be to extend our method with more complex features than the degree in order to find more complex anomalies as well and identify the remaining events unidentified with the degree. This task would be simplified by the fact that largest anomalies have already been removed from the remaining traffic, allowing a more detailed and finer analysis. At broader scale, our work could be useful in the field of IP traffic modelling as we would be able to generate normal traffic according to a specific feature. Likewise, thanks to their individual extraction, anomalies could also be studied separately.

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