Degeneracy-Based Mining of Social and Information Networks: Dynamics and Applications

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Complex Networks Team, LIP6
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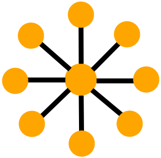
April 1, 2016

Networks

Networks allow to model relationships between entities

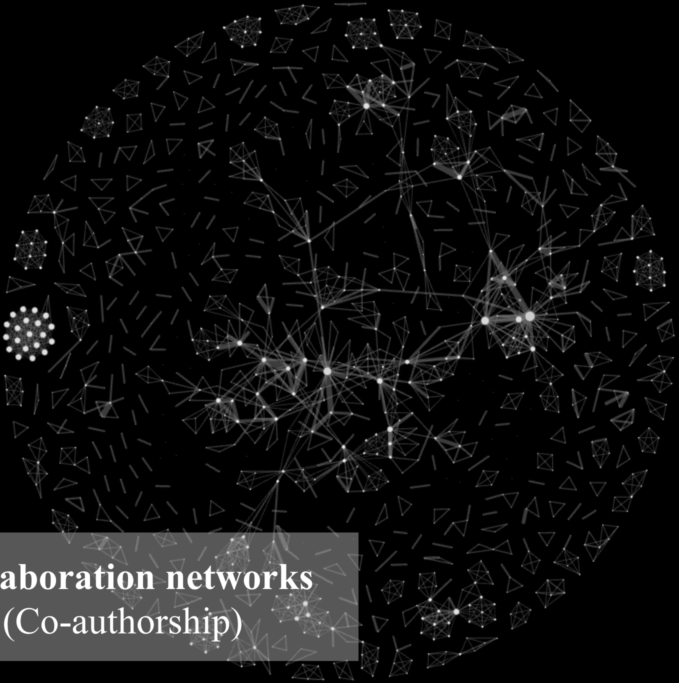


General-purpose language

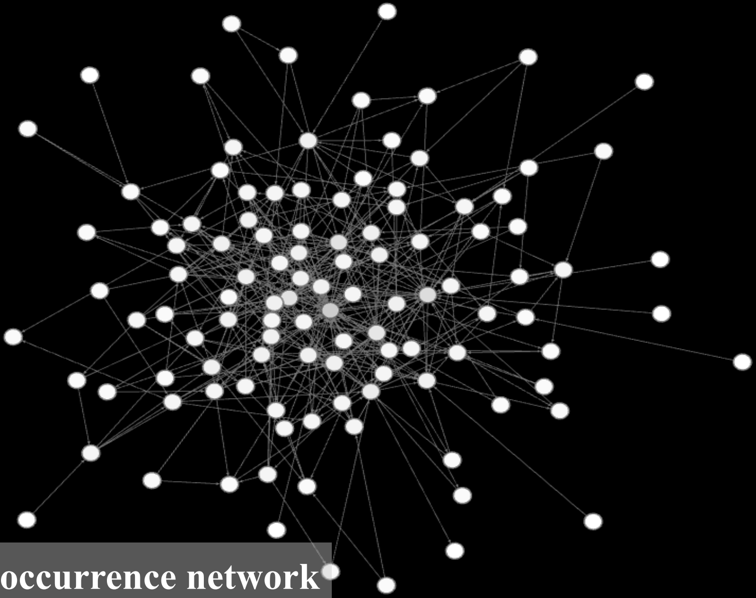
for describing real-world systems

Facebook
1 Billion Users (Aug. 2015)

Source: https://www.facebook.com/zuck



Collaboration networks
(Co-authorship)



Term co-occurrence network
(*David Copperfield* novel by Charles Dickens)

Networks are Everywhere

- **Technological networks:**
  - Internet
  - Telephone networks
  - Power grid
  - Road, airline and rail networks
- **Information networks:**
  - World Wide Web
  - Blog networks
  - Citation networks
  - Textual networks
- **Social networks:**
  - Collaboration networks
  - Organizational networks
  - Communication networks
- **Biological networks:**
  - Neural networks
  - Protein-protein interaction networks
  - Gene regulatory networks
  - Food webs

Motivation: Analytics in the Network Science Era

Study **large real-world** networks

1. Design of effective and efficient **graph mining** algorithms
2. Apply the algorithms to analyze and understand the **structure** and **dynamics** of complex systems
3. Utilize the extracted knowledge to solve **real-world applications**

Contributions

Overview

Models, **tools** and **observations** for problems in the area of mining social and information networks

We build upon **computationally efficient graph mining** methods to:

1. Design models for analyzing the structure and dynamics of real-world networks
   - Unravel properties that can further be used in practical applications
2. Develop algorithmic tools for large-scale analytics on data with:
   - Inherent graph structure (e.g., social networks)
   - Without inherent graph structure (e.g., text)

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Overview

Dynamics of Real Networks

Modeling Social Engagement **[MV, '13]**	Vulnerability Assessment **[MV, '15]**

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Dynamics of Real Networks

Modeling
Social Engagement
[MV, '13]

Vulnerability
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Identification of
Influential Spreaders
[RMV, '15], [MRV, '16]

Graph Clustering
Community Detection
[GMTV, '14]

Algorithmic Tools for Data Analytics

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Degeneracy-based Graph Mining

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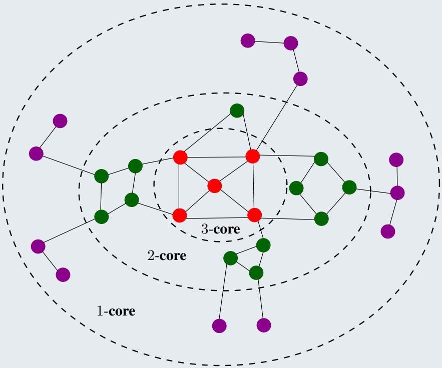
Algorithmic Tools for Data Analytics

k-core Decomposition in Networks

Degeneracy and *k*-core Decomposition

Example



Core number $\mathbf{c_i = 1}$

Core number $\mathbf{c_i = 2}$

Core number $\mathbf{c_i = 3}$

Graph Degeneracy $\delta^*(\mathbf{G}) = \mathbf{3}$

$\mathbf{G_0 = G}$

$\mathbf{G_1} =$ 1-core of $\mathbf{G}$

$\mathbf{G_2} =$ 2-core of $\mathbf{G}$

$\mathbf{G_3} =$ 3-core of $\mathbf{G}$

$\mathbf{G_0 \supseteq G_1 \supseteq G_2 \supseteq G_3}$

[Seidman, '83]

k-core Decomposition: the Algorithm

Algorithm: k-core decomposition

Input: Undirected graph $G = (V, E)$
Output: Core numbers $c(v), \forall v \in V$

```
i ← 0
while |V| > 0 do
    while ∃v : d(v) ≤ i do
        c(v) ← i
        V ← V \ {v}
        E ← E \ {(u, v)|u ∈ V}
    end while
    i ← i + 1
end while
```

More about the decomposition

- Time complexity $\mathcal{O}(|E|)$ [Batagelj and Zaversnik, '03]
- Extensions to various graph types and computation models

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Outline

Modeling Engagement Dynamics in Social Graphs

Modeling Engagement Dynamics

Objectives and contributions

- Goal: model and quantify the engagement in social graphs
- **User engagement** refers to the extend that an individual is encouraged to participate in the activities of a community
- Closely related property to the one of **node departure dynamics**

Main contributions

- Measures of engagement (node and graph level)
- Experiments: Properties and dynamics of real graphs
- Implications of our study on a new problem of robustness/vulnerability assessment

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Problem Statement (1/2)

Goal

Model social engagement from a **network-wise** point of view

- Consider information only about the **underlying graph structure**

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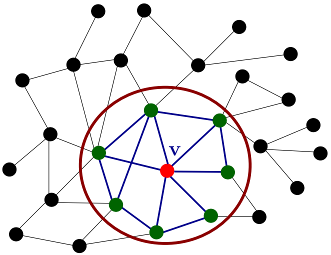
Model social engagement from a **network-wise** point of view

- Consider information only about the **underlying graph structure**

- Each individual that participates in a social activity, derives a **benefit**
  - The benefit emanates from his/her neighborhood
- The benefit of each individual is affected by the degree of interaction among its **neighbors** **[Ugander et al., PNAS '12]**
  - Interactions among friends, can increase user's benefit



Problem Statement (2/2)

- Suppose now that a user decides to **depart**
  - Direct effects in his neighborhood $\longrightarrow$ Some of his friends may also decide to depart
  - A departure can become an **epidemic**, forming a **cascade** of individual departures

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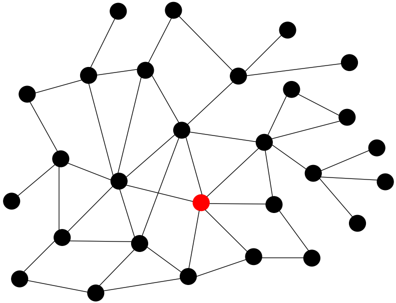

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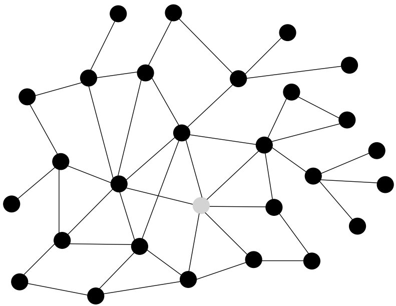

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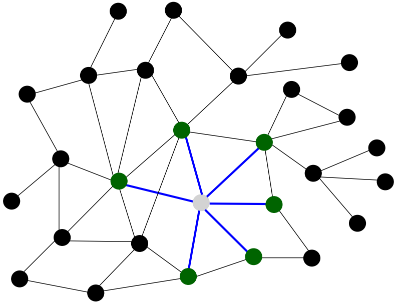

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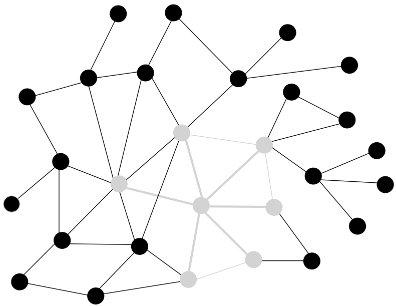

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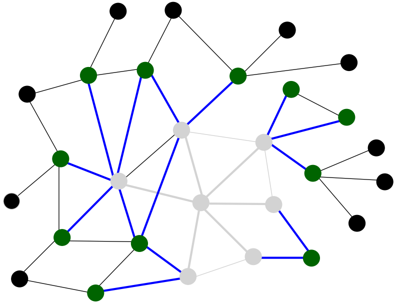

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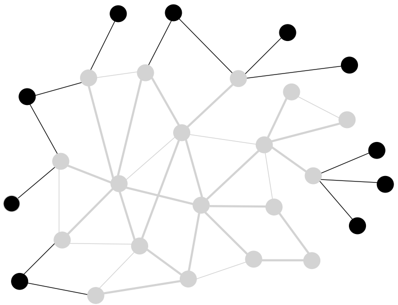

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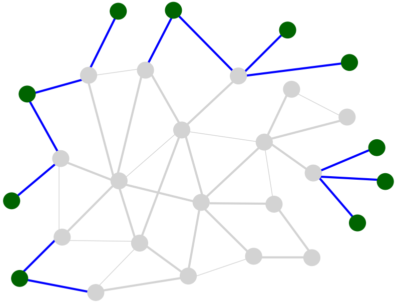

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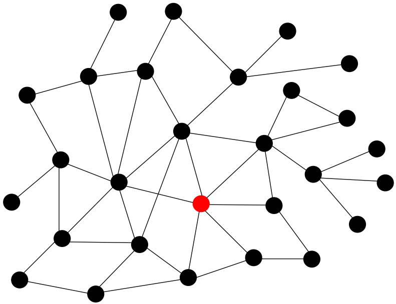

- **Direct-benefit effects:**
  To incur an explicit benefit by remaining engaged

  $\downarrow$

  The decision of a node should align with the one of its neighbors **[Easley and Kleinberg, '10]**

Model Description

- $\mathcal{X} = \{0, 1\}$: set of strategies (i.e., *leave* or *stay*)
  $\mathbf{x} = [x_1, x_2, \ldots, x_n]$: vector that denotes the decision of each node $i \in V$

- *Node payoff* function:
  $\Pi_i(\mathbf{x}) = \textbf{benefit}\left(x_i, \sum_{j \in \mathcal{N}_i} x_j\right) - \textbf{cost}(x_i), \mathcal{N}_i = \{j \in V : (i, j) \in E\}$
  - **Benefit function**: depends on node's own decision and the aggregate decision of the neighbors
  - **Cost function**: does not need to be known a priori $\rightarrow$ remain engaged if cost $\leq$ benefit (non-negative payoff)

Equilibrium Property

The best response of each node $i \in V$ corresponds to the *core number* c_i **[Manshadi and Johari, '09]**, **[Harkins, '13]**, **[Bhawalkar et al., '11]**

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Proposed Engagement Measures (1/3)

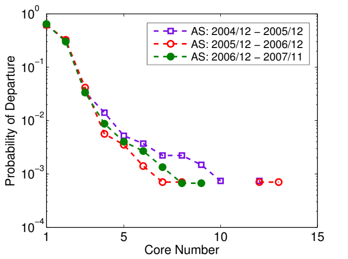
Node engagement

Proposition (Node Engagement)

The engagement level e_i of each node $i \in V$ is defined as its core number c_i

- Nodes with higher core number $\longrightarrow$ better engagement

Prob. of departure vs. core number



CAIDA graph

- More refined modeling explanation of the **departure dynamics** in social graphs **[Wu et al., '13]**
  - **Active users:** core of the graph
  - **Inactive users:** periphery of the graph
  - The departure of nodes is *proportional* to their position in the graph

Proposed Engagement Measures (2/3)

Engagement subgraphs

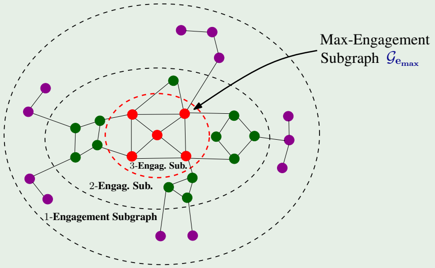
Definition (k-Engagement Subgraph $\mathcal{G}_k$)

The graph which is induced by the nodes $i \in V$ with engagement level $e_i \geq k$

Proposition (Max-Engagement Subgraph $\mathcal{G}_{e_{\max}}$)

- Maximum engagement level of the graph: $e_{\max} = \delta^*(G)$ (degeneracy of the graph)
- **Max-Engagement subgraph**: induced by the nodes with engagement $e = e_{\max}$

Example graph



Proposed Engagement Measures (3/3)

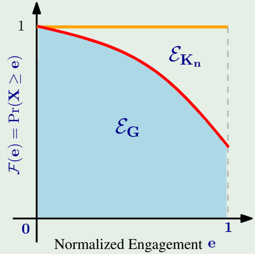
Graph engagement

Definition (**Graph Engagement $\mathcal{E}_G$**)

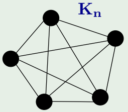
The total engagement level of a graph G is defined as the area under curve of the normalized CCDF $\mathcal{F}(e) = \Pr(X \geq e)$

- Values in the range $[0, 1]$
- Higher $\mathcal{E}_G$ values $\rightarrow$ higher total engagement

Schematic representation of $\mathcal{E}_G$



Complete graph



Datasets

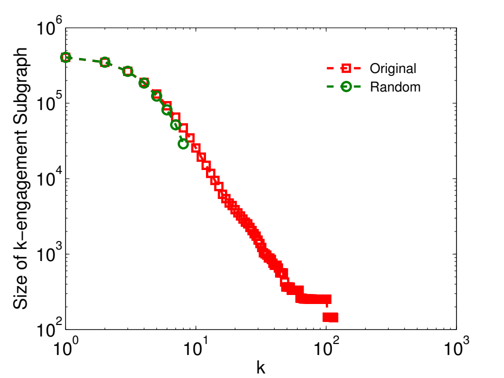
Basic characteristics of real-world networks

Graph	# Nodes	# Edges
FACEBOOK	63,392	816,886
YOUTUBE	1,134,890	2,987,624
SLASHDOT	77,360	546,487
EPINIONS	75,877	405,739
EMAIL-EUALL	224,832	340,795
EMAIL-ENRON	33,696	180,811
CA-GR-QC	4,158	13,428
CA-ASTRO-PH	17,903	197,031
CA-HEP-PH	11,204	117,649
CA-HEP-TH	8,638	24,827
CA-COND-MAT	21,363	91,342
DBLP	404,892	1,422,263

Properties of *k*-Engagement Subgraphs

Size distribution

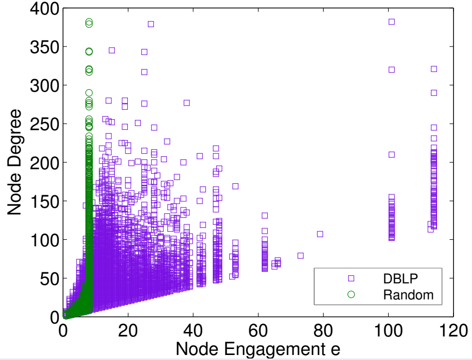
DBLP



- Almost **skewed** distribution
  - Small subgraphs with high engagement ***k***
- Different behavior between **real** and **randomly** rewired graphs

Node Engagement vs. Node Degree

DBLP

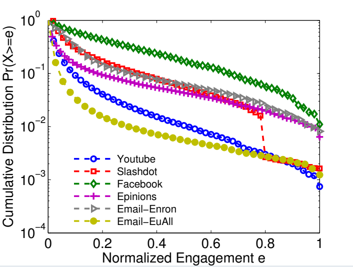


High degree nodes are possible to have low engagement

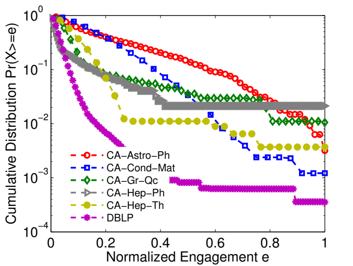
Graphs' Engagement Properties

Engagement index $\mathcal{E}_G$

$\mathcal{E}_G$ index for social graphs



$\mathcal{E}_G$ index for collaboration graphs



- FACEBOOK has the maximum engagement index $\mathcal{E}_G$
  - A relatively high fraction of nodes has high (normalized) engagement e
- DBLP shows the lower engagement index $\mathcal{E}_G$ in the collaboration graphs
  - **Possible explanation:** significant number of "relatively" new authors with low engagement

Discussion

- Engagement metrics and dynamics in social graphs
  - Local and global metrics
  - k-core decomposition-based
  - Limitation: Ground truth data to further evaluate the proposed metrics
- Engagement-based vulnerability assessment

Vulnerability Assessment in Social Networks under Cascade-Based Node Departures

Vulnerability Assessment in Real Networks

- Well-studied problem in the broad area of network science
  - **[Albert and Barabási, Rev. Mod. Phys. '02]**
- Networks with **skewed degree distribution**
  - **Robust** against random failures
  - **Vulnerable** under targeted attacks to hubs
  - **[Albert et al., Nature '00]**

Engagement-based Vulnerability Assessment

Motivation of this work

- In social networks
  - Instead of **degree-based** failures and attacks ...
  - Departures based on the **engagement** level
- The engagement level of nodes is not accurately captured by the node degree
- Well-known **degree-based** notions of robustness assessment may not accurately capture this feature of social networks
  - **[Albert et al., Nature '00]**
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Contributions of this Work

Goal: novel concept of **engagement-based vulnerability assessment** in social graphs

- Cascading Departure (CasD) model
  - Captures the cascading disengagement effect due to the departure of a node
- Vulnerability assessment under node departures
  - Experimental results on real networks

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Disengagement Epidemic Model

How to model user departures – the idea

- We introduce a model of **cascading departures** in social graphs
- Main idea:
  - The departure of a node can cause direct effects in its neighborhood
  - Some of the neighbors may also decide to depart
  - It can lead to an **epidemic of disengagement** (or churn effect)
  - Model the process using the $\boldsymbol{k}$-core decomposition and its connection to the engagement dynamics

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Cascading Departure (CasD) Model

Algorithm: CasD model

Input: Undirected graph $G = (V, E)$ and node $v \in V$
Output: Set of removed nodes R

```
c = [c_1, c_2, ..., c_|V|] = k-core_decomposition(G)
Ṽ = V \ {v} {Remove node v and the incident edges}
repeat
    {Recompute the core number of each node i ∈ Ṽ}
    c̃ = k-core_decomposition(G)
    {Normalize the core numbers c̃_i into the interval [0, 1]}
    c̃_i^norm = (c̃_i − min(c̃)) / (max(c̃) − min(c̃)), ∀i ∈ Ṽ
    for all i ∈ Ṽ do
      if c̃_i < c_i then
        Remove node i from G with probability:
        Pr(Ṽ = Ṽ \ {i}) = 1 − c̃_i^norm   (also R = R ∪ {i})
      end if
    end for
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Cascading Departure (CasD) Model

Algorithm: CasD model

Input: Undirected graph $G = (V, E)$ and node $v \in V$

Output: Set of removed nodes R

$\mathbf{c} = [c_1, c_2, \ldots, c_{|V|}] = \texttt{k-core\_decomposition}(G)$

$\tilde{V} = V \setminus \{v\}$ {Remove node v and the incident edges}

repeat

  {Recompute the core number of each node $i \in \tilde{V}$}

  $\tilde{\mathbf{c}} = \texttt{k-core\_decomposition}(G)$

  {Normalize the core numbers $\tilde{c}_i$ into the interval $[0, 1]$}

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Social Vulnerability Assessment

Our approach

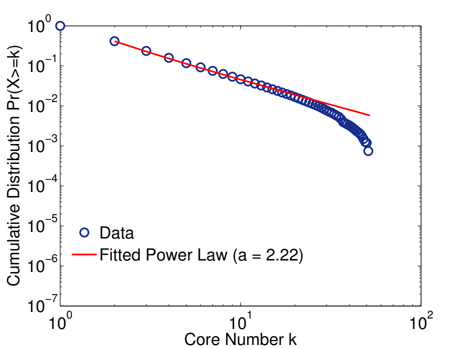
- **Vulnerability assessment** concept combining

  1. The CasD model (how the disengagement epidemic is spreading)
  2. Observation: skewness of the core number distribution

Observations on Real Graphs

Core number distribution

YOUTUBE



- Heavy-tailed core size distribution
- Most of the nodes typically have low core number $\rightarrow$ low engagement
- Randomly selected node $\rightarrow$ more probable to have **low core number**

Vulnerability Assessment under Node Departures

Experimental set-up

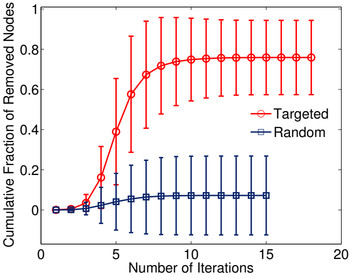
Two strategies for selecting the node that will depart first:

1. **Random departure**: a randomly selected node leaves the graph
   - What is more probable to occur
2. **Targeted departure**: a node selected among the ones with the highest core number decides to depart
   - Highly engaged individuals disengage

CasD Model: Application on Real Graphs

Cumulative fraction of removed nodes

FACEBOOK



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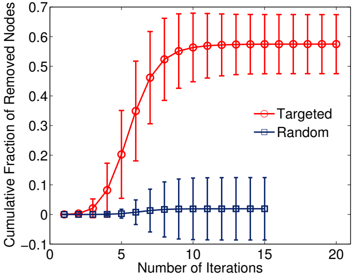

- Cumulative fraction of removed nodes during the execution of the model for **random** and **targeted** departures

CasD Model: Application on Real Graphs

Cumulative fraction of removed nodes

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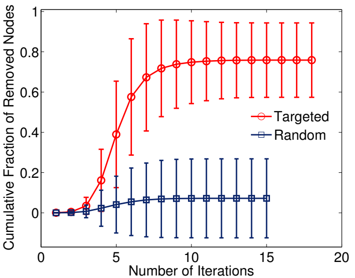

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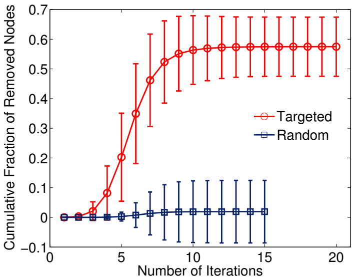

- Cumulative fraction of removed nodes during the execution of the model for **random** and **targeted** departures

- **Robustness** against cascades triggered by **random departures** of nodes
- **Vulnerability** under cascades triggered by **targeted departures** of high core (engaged) nodes

Discussion

- New concept of **vulnerability assessment** in social networks
  - User departures instead of failures and attacks
  - Engagement-based instead of degree-based
- ***k***-core decomposition-based model
- Departure of highly engaged nodes can trigger big cascade

Locating Influential Spreaders in Social Networks

Identification of Influential Spreaders

Motivation

- **Spreading processes** in complex networks
  - Spread of news and ideas
  - Diffusion of influence
  - Disease propagation
  - Viral marketing (word-of-mouth effect)
  - ...
- Identification of **influential spreaders** (**goal of this work**)
  - Able to diffuse information to a large part of the network
  - Understand and control spreading dynamics
    - E.g, vaccinate individuals with good spreading properties in epidemic control

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Identification of Influential Spreaders: the Process

Typically, a two-step approach:

1. Consider a **topological** or **centrality** criterion of the nodes
   - Rank the nodes accordingly
   - The top-ranked nodes are candidates for the most influential ones
2. Simulate the spreading process over the network to examine the performance of the chosen nodes

[Pei and Makse, '13]

Identification of Single Influential Spreaders

Related work

- **Straightforward approach:** consider **degree centrality**
  - High degree nodes are expected to be good spreaders
  - Hub nodes can trigger big cascades

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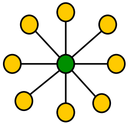
  - Bad case: star subgraph



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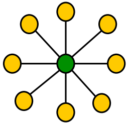
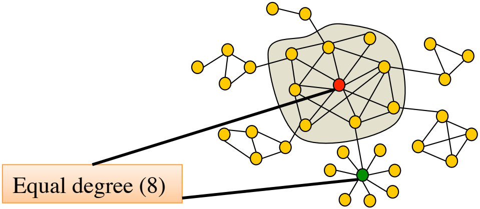
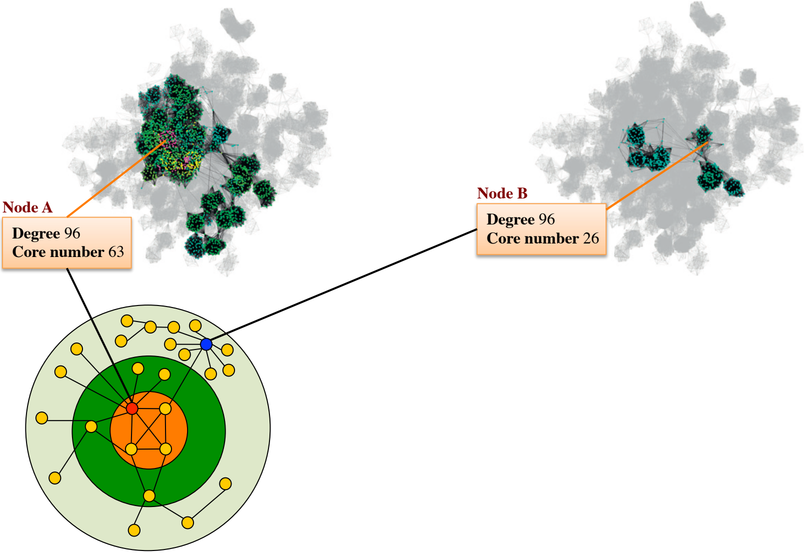


- **Core-periphery structure** of real-world networks



The *k*-core Decomposition Finds Good Spreaders



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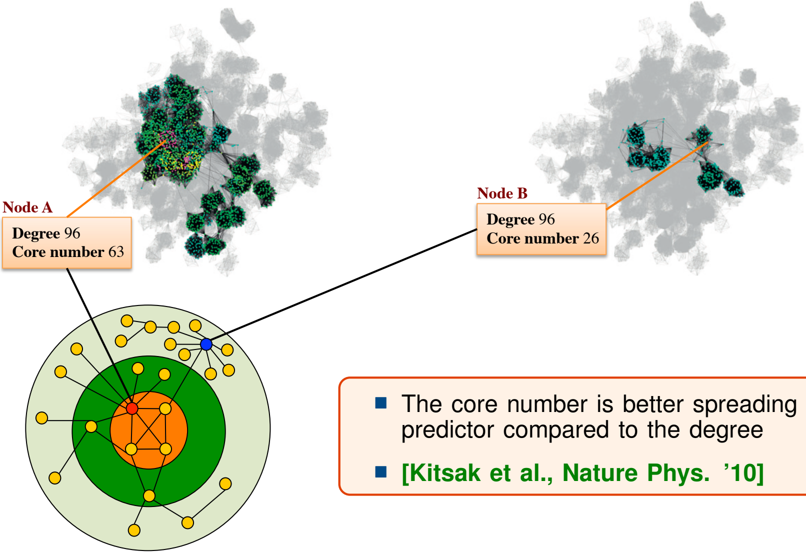

- The core number is better spreading predictor compared to the degree
- [Kitsak et al., Nature Phys. '10]

Motivation and Contributions

- The ***k***-core decomposition often returns a relatively large number of candidate influential spreaders
  - Only a small fraction corresponds to highly influential nodes
- Can we further **refine** the set of the most influential spreaders?

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Main contributions

- Propose the K-truss decomposition for locating influential nodes
  - Triangle-based extension of the k-core decomposition
- Experimental evaluation
  - Better spreading behavior
  - Faster and wider epidemic spreading

K-truss Decomposition (1/2)

Definitions

K-truss subgraph T_K [Cohen, '08], [Wang and Cheng, '12]

K-truss $T_K = (V_{T_K}, E_{T_K}), K \geq 2$: the largest subgraph of G where every edge is contained in at least $K - 2$ triangles within the subgraph

Maximal K-truss subgraph

- The K-truss subgraph defined for the maximum value K_{max} of K
- The nodes of this subgraph define set $\mathcal{T}$

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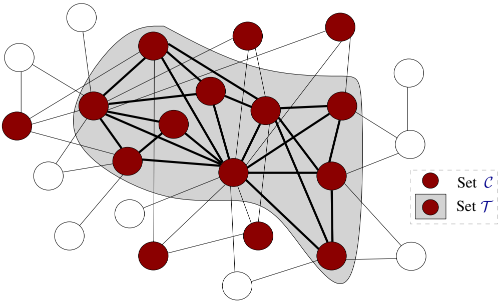
K-truss Decomposition (2/2)

Maximal k-core and K-truss subgraphs

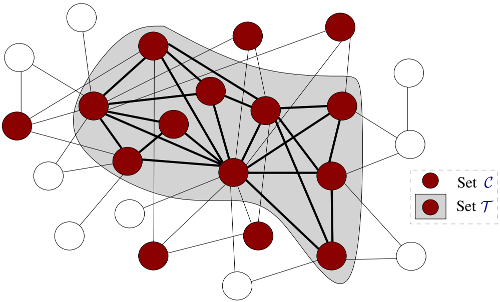


- Set $\mathcal{C}$: nodes of maximal k-core
  - **3**-core subgraph
- Set $\mathcal{T}$: nodes of maximal K-truss
  - **4**-truss subgraph

- The maximal k-core and K-truss subgraphs overlap
  - K-truss is a subgraph of k-core (**core** of the k-core)
  - Heuristic to improve execution time
- We argue that set $\mathcal{T}$ contains **highly influential** nodes

K-truss Decomposition (2/2)

Maximal *k*-core and *K*-truss subgraphs



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Experimental Set-up (1/2)

Baseline methods

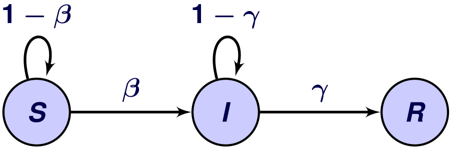
- **truss** method: nodes belonging to set $\mathcal{T}$ (**Proposed method**)
- **core** method: nodes belonging to set $\mathcal{C} - \mathcal{T}$
- **top degree** method: nodes with highest degree $\mathcal{T}$
  - Choose $|\mathcal{C}| - |\mathcal{T}|$ nodes for fair comparison

Experimental Set-up (2/2)

How to simulate the spreading process?

Susceptible-Infected-Recovered (SIR) model

1. Set candidate node as infected (*I* state)
2. An infected node can infect its susceptible neighbors with probability β
   - Set β close to the epidemic threshold $\tau = \frac{1}{\lambda_1}$ **[Chakrabarty et al., '08]**
3. An infected node can recover (stop being active) with probability γ
   - Set $\gamma = \mathbf{0.8}$
4. Count the total number of infected individuals (avg. over multiple runs)



State diagram of the SIR model

[Barrat et al., '08]

Datasets and Properties

Characteristics of the K-truss subgraphs

Network Name	Nodes	Edges	k_{max}	K_{max}	$\|\mathcal{C}\|$	$\|\mathcal{T}\|$	τ
EMAIL-ENRON	33,696	180,811	43	22	275	45	0.00840
EPINIONS	75,877	405,739	67	33	486	61	0.00540
WIKI-VOTE	7,066	100,736	53	23	336	50	0.00720
EMAIL-EUALL	224,832	340,795	37	20	292	62	0.00970
SLASHDOT	82,168	582,533	55	36	134	96	0.00074
WIKI-TALK	2,388,953	4,656,682	131	53	700	237	0.00870

- $\tau = 1/\lambda_1$: epidemic threshold of the graph (λ_1: largest eigenvalue of $\mathbf{A}$)
- Set $\mathcal{T}$ has significantly smaller size compared to set $\mathcal{C}$

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Evaluation the Spreading Process

Apply SIR simulation starting from a single node v each time

- Number of infected nodes at each time step of the process
- Total number of infected nodes M_v
- The time step where the epidemic fades out

Average Number of Infected Nodes

		Time Step							
	Method	**2**	**4**	**6**	**8**	**10**	*Final step*	σ	*Max step*
EMAIL-ENRON	**truss**	**8.44**	**46.66**	**204.08**	**418.77**	**355.84**	**2,596.52**	**136.7**	**33**
	core	**4.78**	**31.97**	**152.55**	**367.28**	**364.13**	**2,465.60**	**199.6**	**37**
	top degree	**6.89**	**34.13**	**155.48**	**360.89**	**357.08**	**2,471.67**	**354.8**	**36**
EPINIONS	**truss**	**4.17**	**19.70**	**75.04**	**204.14**	**329.08**	**2,567.69**	**227.8**	**37**
	core	**3.45**	**14.72**	**55.27**	**158.56**	**280.03**	**2,325.37**	**327.2**	**43**
	top degree	**4.22**	**16.03**	**58.84**	**166.23**	**289.49**	**2,414.99**	**331.7**	**47**
WIKI-VOTE	**truss**	**2.92**	**6.92**	**15.27**	**28.73**	**42.46**	**560.66**	**114.9**	**52**
	core	**1.92**	**4.78**	**10.65**	**20.66**	**32.40**	**466.01**	**104.5**	**57**
	top degree	**2.43**	**5.46**	**12.05**	**23.05**	**35.55**	**502.88**	**104.5**	**62**
EMAIL-EUALL	**truss**	**11.62**	**62.25**	**240.97**	**584.87**	**725.42**	**5,018.52**	**487.94**	**36**
	core	**9.85**	**40.82**	**158.72**	**433.81**	**644.76**	**4,579.84**	**498.71**	**38**
	top degree	**17.96**	**39.93**	**144.69**	**503.18**	**548.25**	**4,137.56**	**1,174.84**	**39**

- Performance of **truss** method
  - **Higher** infection rate during the first steps of the process
  - The total number of infected nodes is **larger**
  - The epidemic dies out **earlier**

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	core	3.45	14.72	55.27	158.56	280.03	2,325.37	327.2	43
	top degree	4.22	16.03	58.84	166.23	289.49	2,414.99	331.7	47
WIKI-VOTE	truss	2.92	6.92	15.27	28.73	42.46	560.66	114.9	52
	core	1.92	4.78	10.65	20.66	32.40	466.01	104.5	57
	top degree	2.43	5.46	12.05	23.05	35.55	502.88	104.5	62
EMAIL-EUALL	truss	11.62	62.25	240.97	584.87	725.42	5,018.52	487.94	36
	core	9.85	40.82	158.72	433.81	644.76	4,579.84	498.71	38
	top degree	17.96	39.93	144.69	503.18	548.25	4,137.56	1,174.84	39

- Performance of **truss** method
  - **Higher** infection rate during the first steps of the process
  - The total number of infected nodes is **larger**
  - The epidemic dies out **earlier**

Average Number of Infected Nodes

	Method	2	4	6	8	10	*Final step*	σ	*Max step*
		Time Step							
EMAIL-ENRON	truss	8.44	46.66	204.08	418.77	355.84	2,596.52	136.7	33
	core	4.78	31.97	152.55	367.28	364.13	2,465.60	199.6	37
	top degree	6.89	34.13	155.48	360.89	357.08	2,471.67	354.8	36
EPINIONS	truss	4.17	19.70	75.04	204.14	329.08	2,567.69	227.8	37
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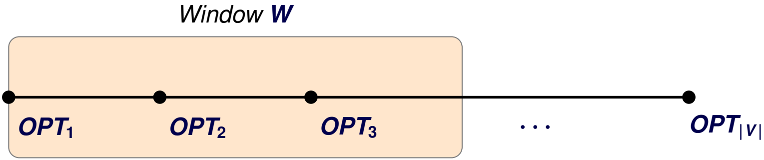
Comparison to the Optimal Spreading (1/2)

Methodology



1. Rank nodes according to the total infection size M_v
   - $OPT_1 \geq OPT_2 \geq \ldots \geq OPT_{|V|}$, where $OPT_1 = \arg\max_{v \in V} \; M_v$
2. Consider window W over the ranked nodes

$$P_W^{\mathcal{T}} = \frac{|T_W|/|\mathcal{T}|}{|W|/|V|}$$

   - T_W is the set of nodes $v \in \mathcal{T}$ located in W

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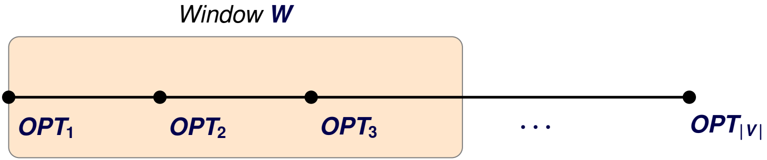

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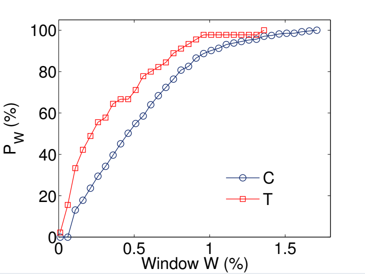
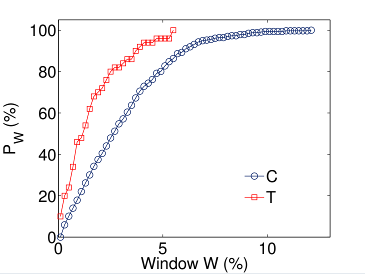
Comparison to the Optimal Spreading (2/2)

Results

EMAIL-ENRON



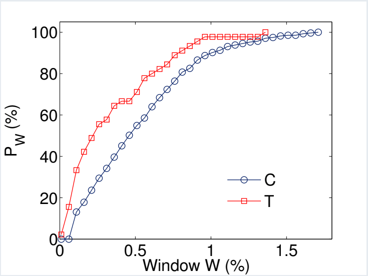
WIKI-VOTE



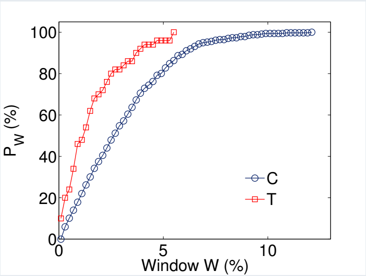
Comparison to the Optimal Spreading (2/2)

Results

EMAIL-ENRON



WIKI-VOTE



- $\boldsymbol{P_W^{\mathcal{T}}}$ reaches the maximum value (i.e., **100%**) relatively early and for small window sizes, compared to $\boldsymbol{P_W^{\mathcal{C}}}$
- The nodes detected by the $\boldsymbol{K}$-truss decomposition are better distributed among the most efficient spreaders

Discussion

- The K-truss decomposition can help towards identifying single influential spreaders
  - Faster and wider epidemic spreading
  - Well distributed nodes among those that are achieving the optimal spreading

Concluding Remarks

Degeneracy-based Graph Mining

Graph mining and **core decomposition**

1. Models, dynamics and properties of social networks
2. Algorithmic tools for graph analytics

Summary of Contributions

C1: ***k***-core decomposition for modeling the engagement dynamics

C2: Model for vulnerability assessment under node departures in social networks

C3: The ***K***-truss decomposition method locates highly influential nodes

C4: Accelerating graph clustering and community detection with the ***k***-core decomposition

C5: Graph mining and core decomposition for text analytics

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Future Work

- Engagement dynamics on graphs with rich semantics
  - What about ground truth information?
- Prediction algorithms for network vulnerability
- Identification of multiple influential spreaders

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F. D. Malliaros and M. Vazirgiannis
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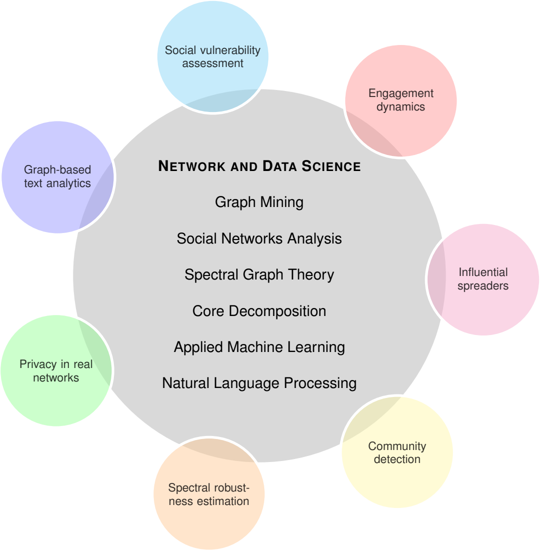
Thank You!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NETWORK AND DATA SCIENCE
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Influential spreaders
Privacy in real networks
Community detection
Spectral robustness estimation