

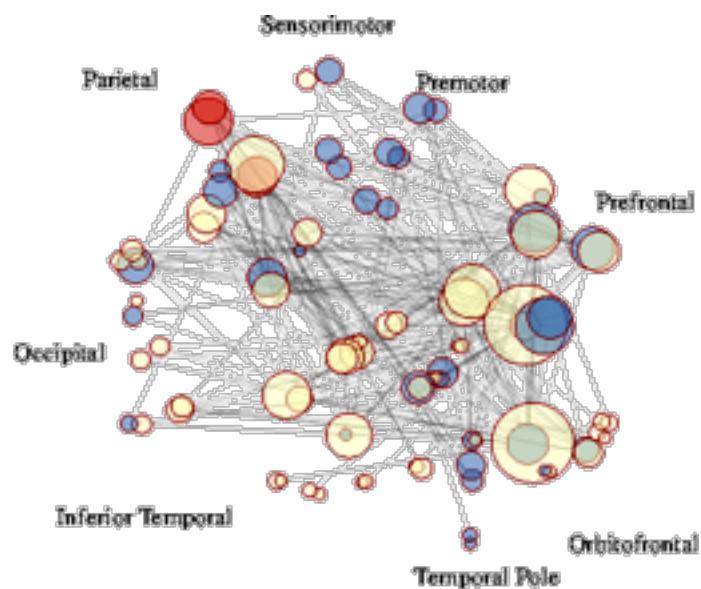
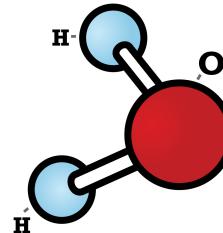


# Graph analysis of functional brain networks: theory, applications and issues

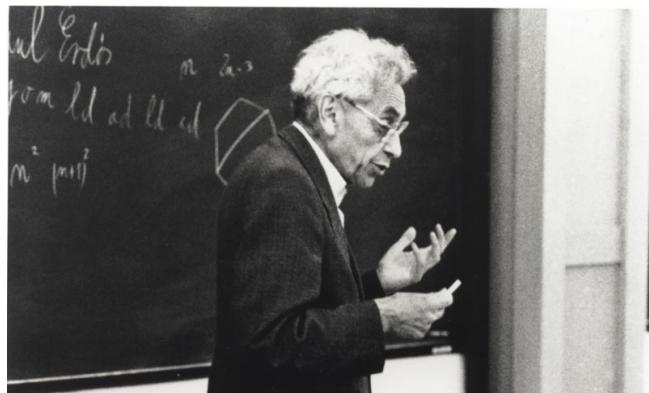
Fabrizio DE VICO FALLANI

INRIA Paris-Rocquencourt – ARAMIS team  
Institut du Cerveau et de la Moelle épinière (ICM)  
Paris, France

# Emergence in connected systems



# A graph theoretical approach



Paul Erdos (1913-1996)

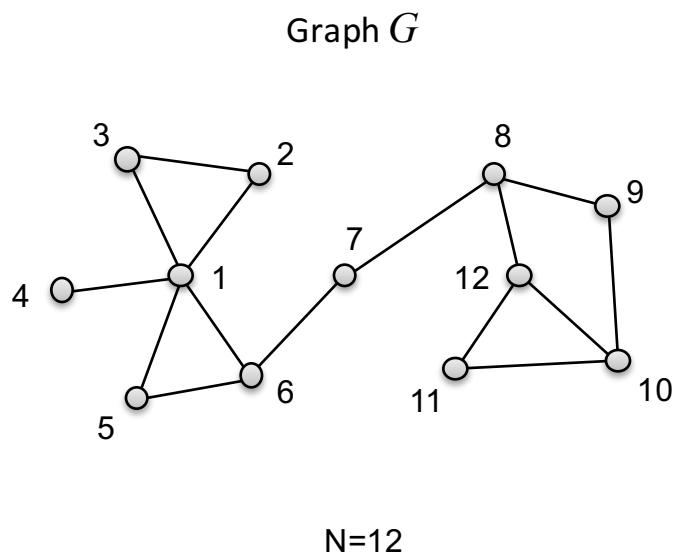
Graduate Texts  
in Mathematics

Béla Bollobás

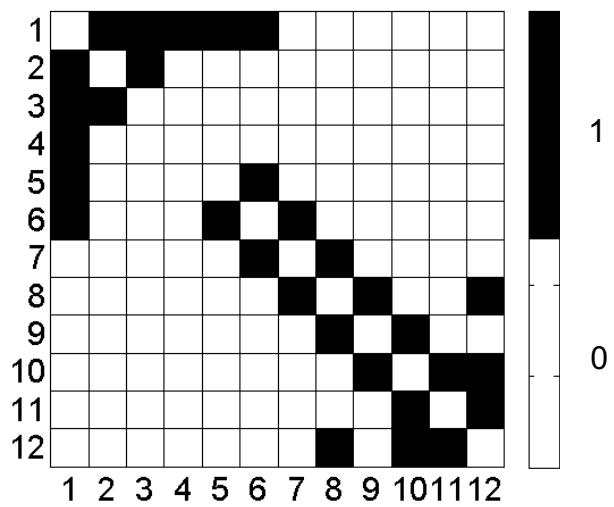
Modern  
Graph Theory



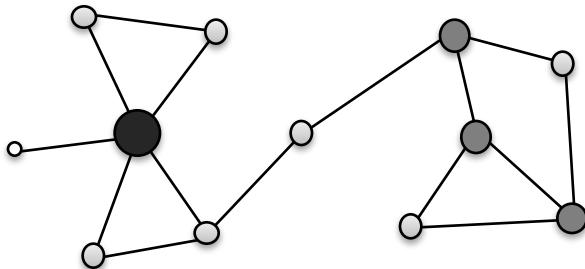
1998



$$\mathbf{A} = \{a_{i,j}\} =$$

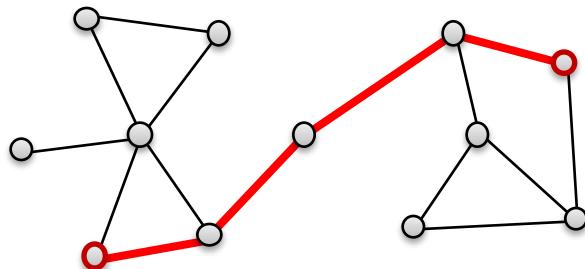


# Quantifying network properties



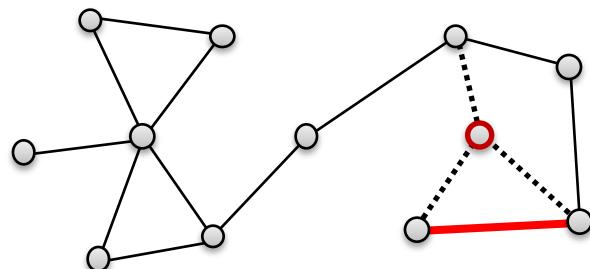
**Node degree**

$$k(i) = \sum_{j=1}^N a_{i,j}$$



**Global efficiency (1/distance)**

$$E_{\text{glo}} = \frac{1}{N(N-1)} \sum_{i,j=1}^N \frac{1}{d_{i,j}}$$

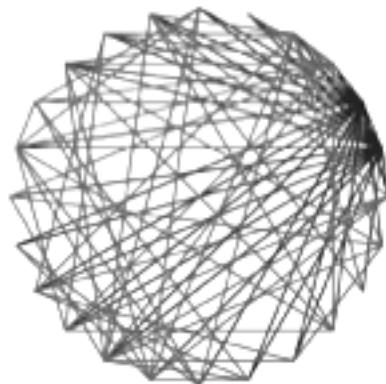


**Clustering (Local efficiency)**

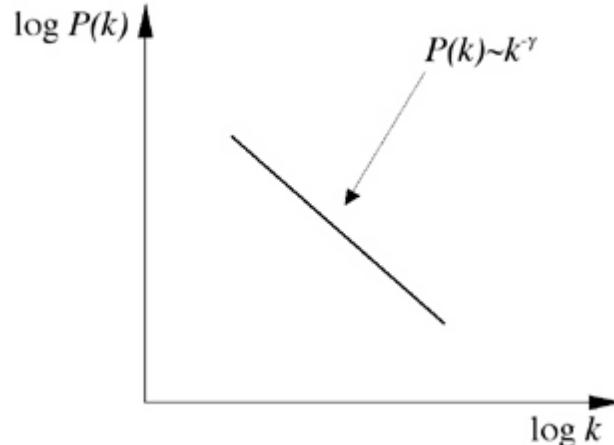
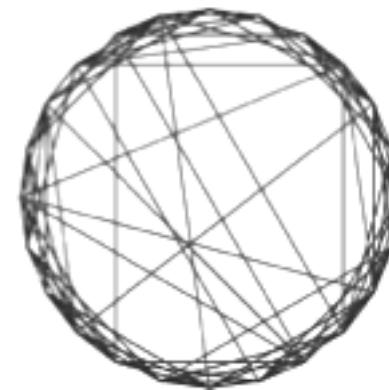
$$E_{\text{loc}} = \frac{1}{N} \sum_{i=1}^N E_{\text{glob}}(i)$$

# Universal properties of complex networks

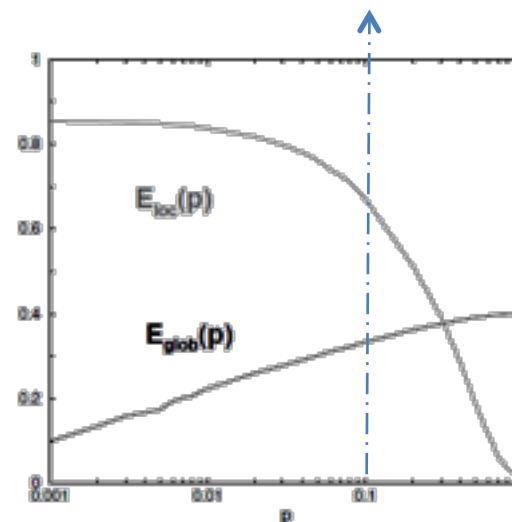
Scale-free



Small-world

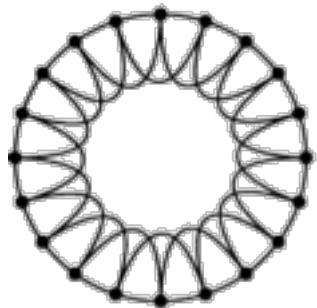


Barabasi & Albert, Nature, 2001

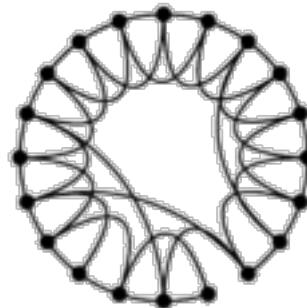


Watts & Strogatz, Science, 1998

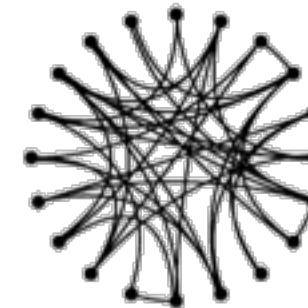
# Small-world brain networks



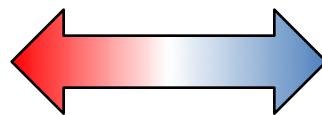
Regular



Small-world

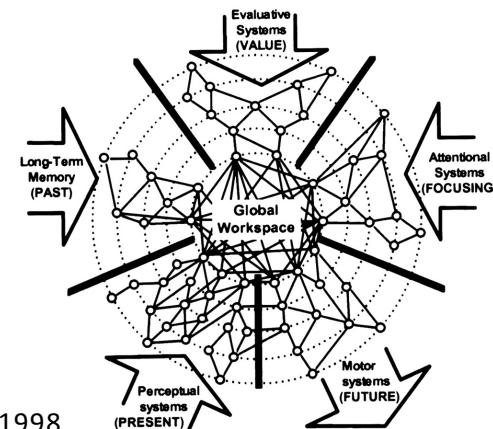


Random



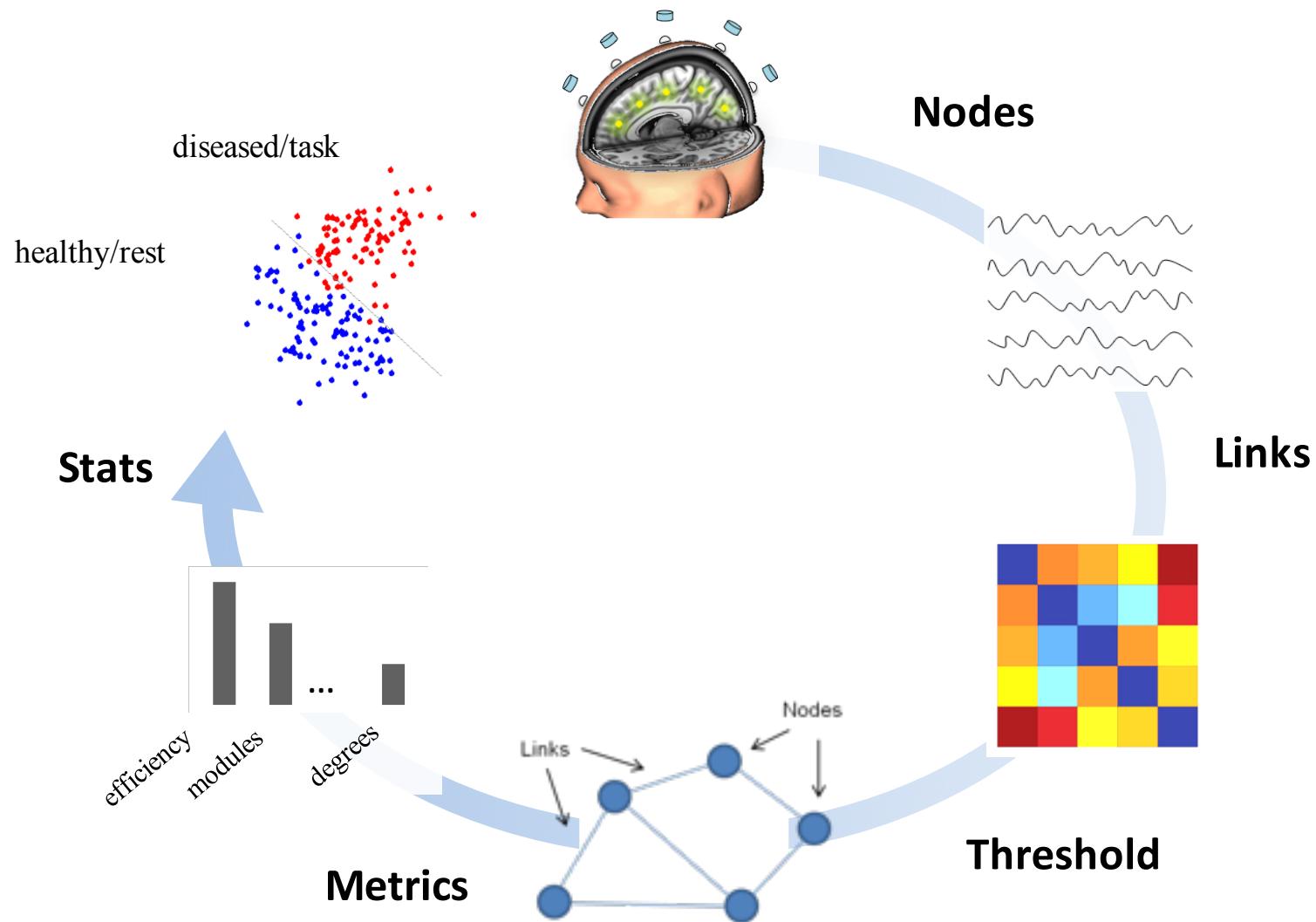
Segregation

Dahene et al, PNAS, 1998

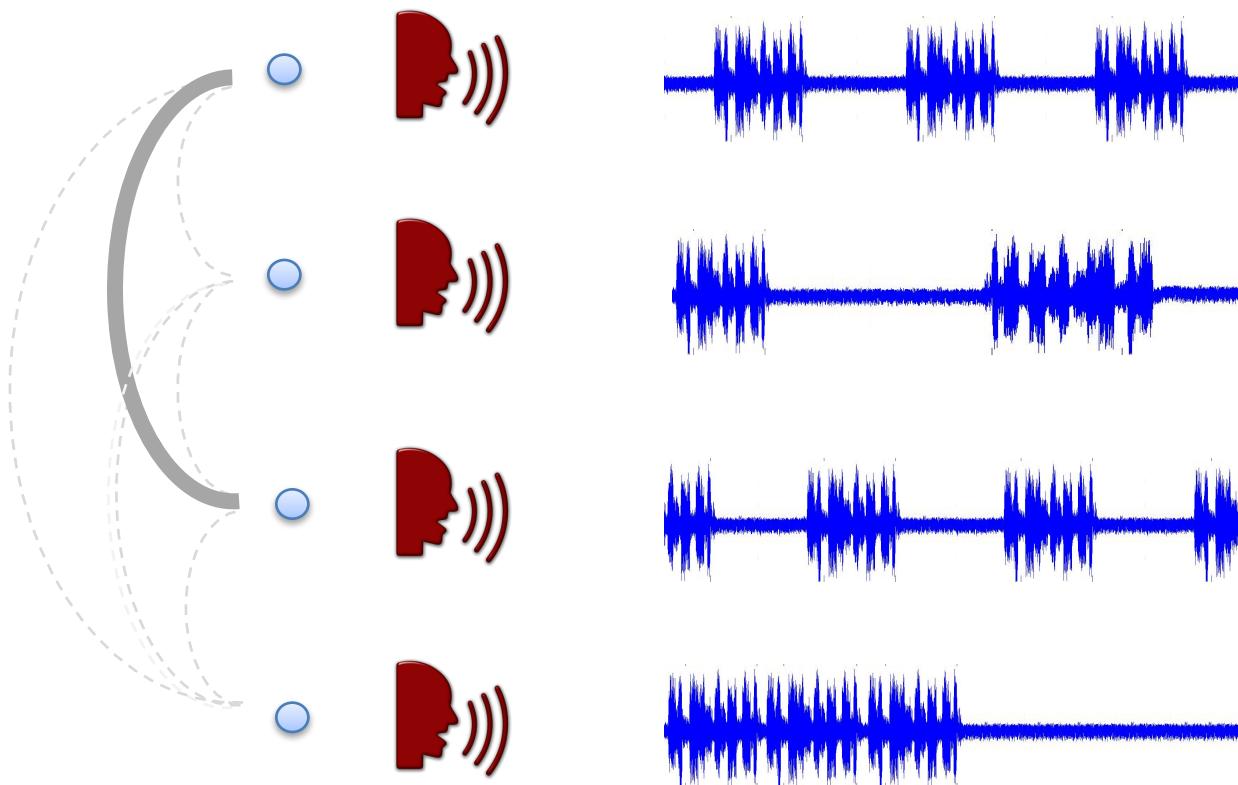


Integration

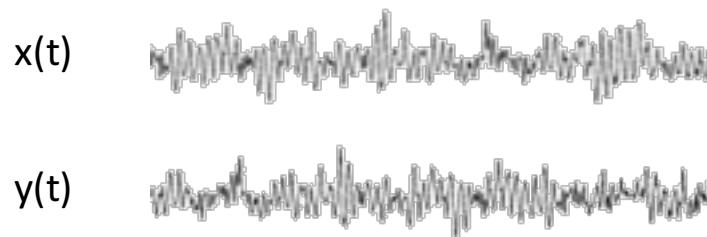
# Graph analysis of functional brain networks



# Inferring connectivity from signals

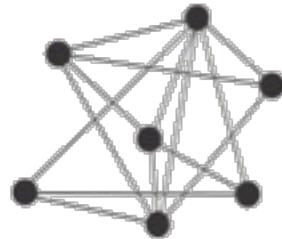


# Functional connectivity



**Normalized cross-covariance**

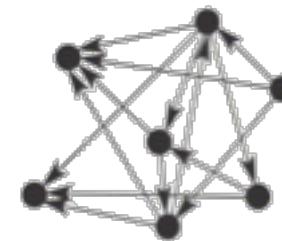
$$C(x, y) = \frac{\sum_{t=1}^T (x(t) - \mu_x)(y(t) - \mu_y)}{\sigma_x \sigma_y}$$



Synchronization (undirected)

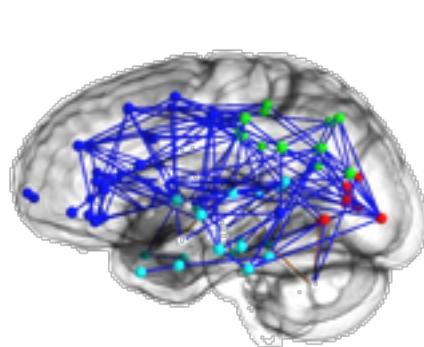
**Granger-causality** (AR modeling)

$$G_{x \rightarrow y} = \ln\left(\frac{\text{var}(e_y)}{\text{var}(e_{xy})}\right)$$

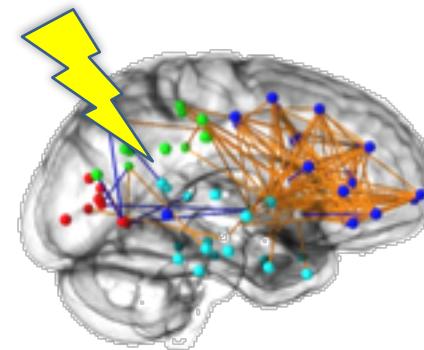


Propagation (directed)

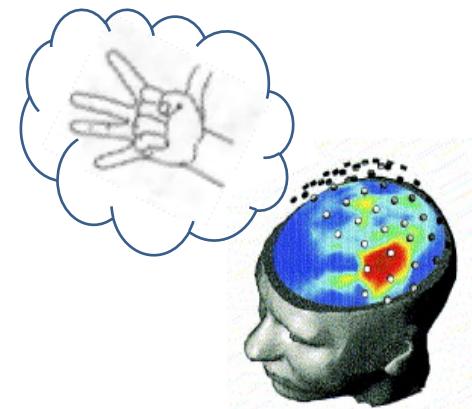
# Cortical reorganization after stroke



**Disability**



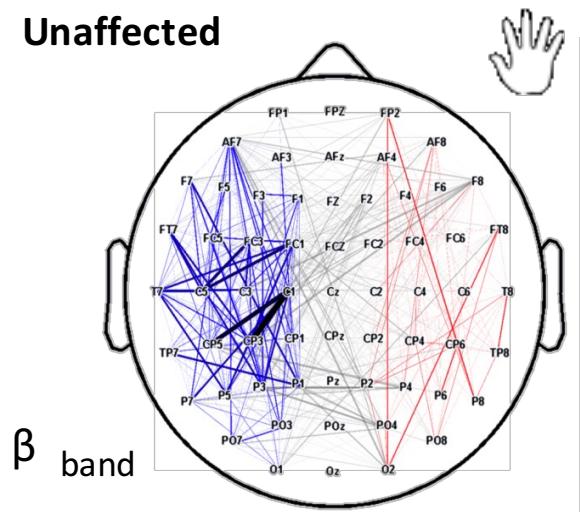
**Motor Imagery**



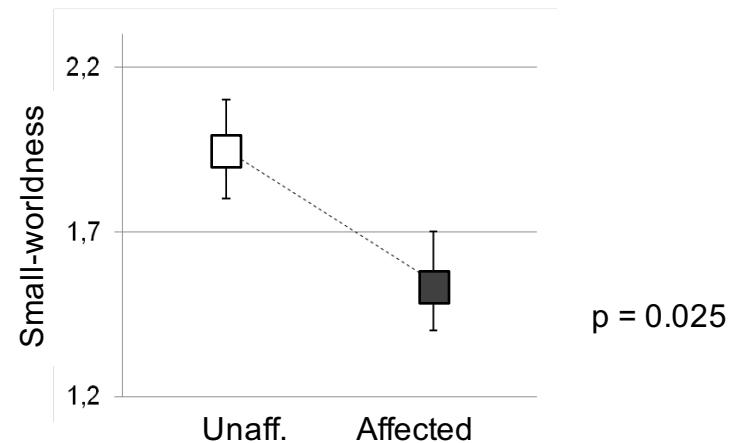
(Pfurtscheller and Neuper, Neurosci Lett, 1997)

# Reduced network efficiency and outcome prediction (Macroscale)

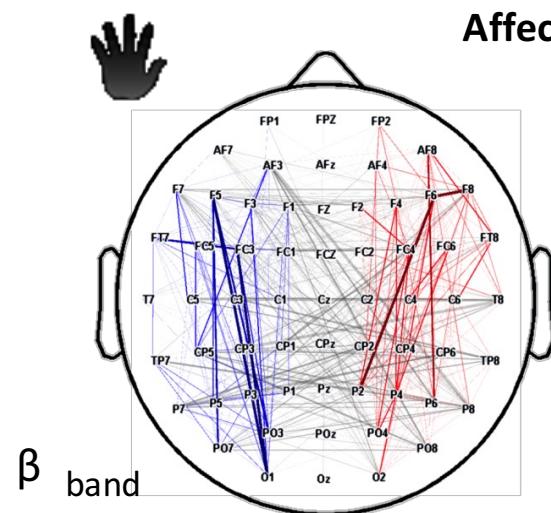
Unaffected



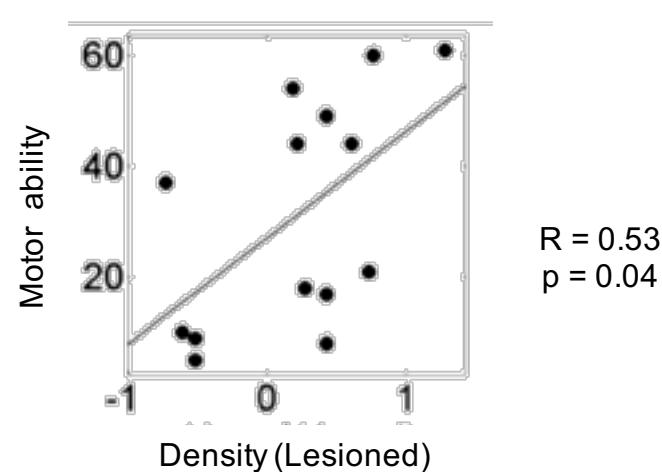
$\beta$  band



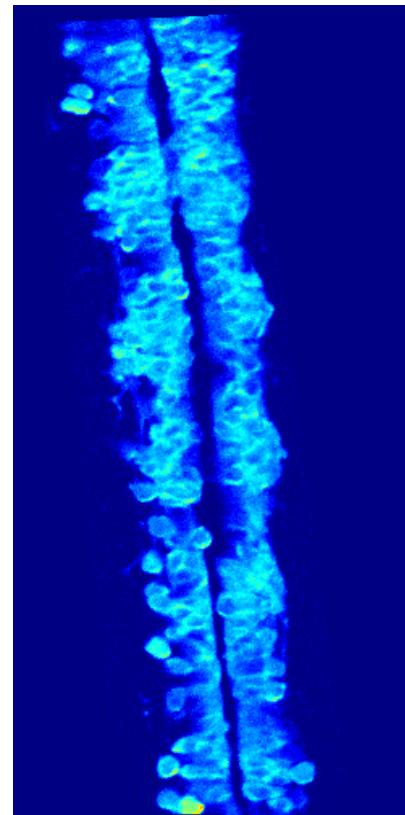
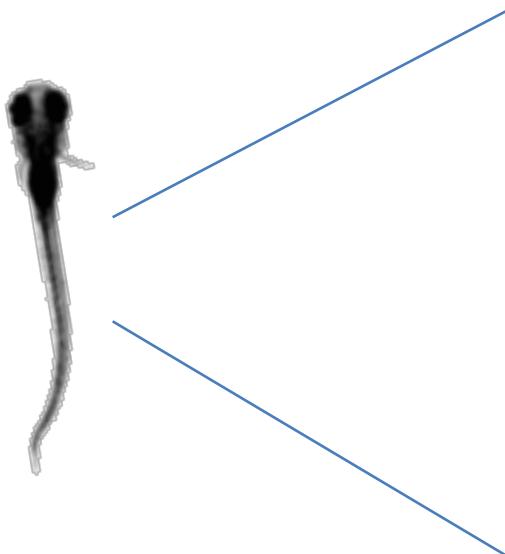
Affected



$\beta$  band

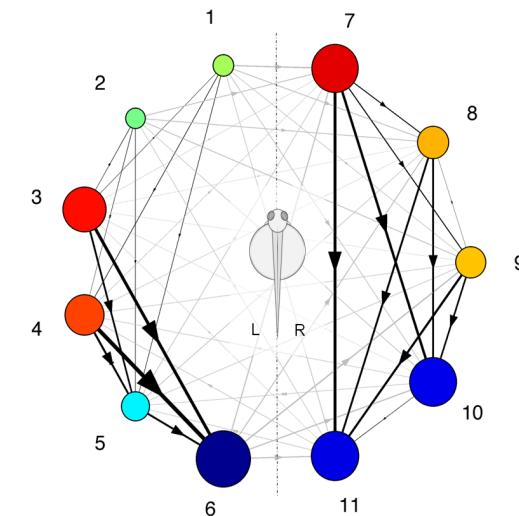
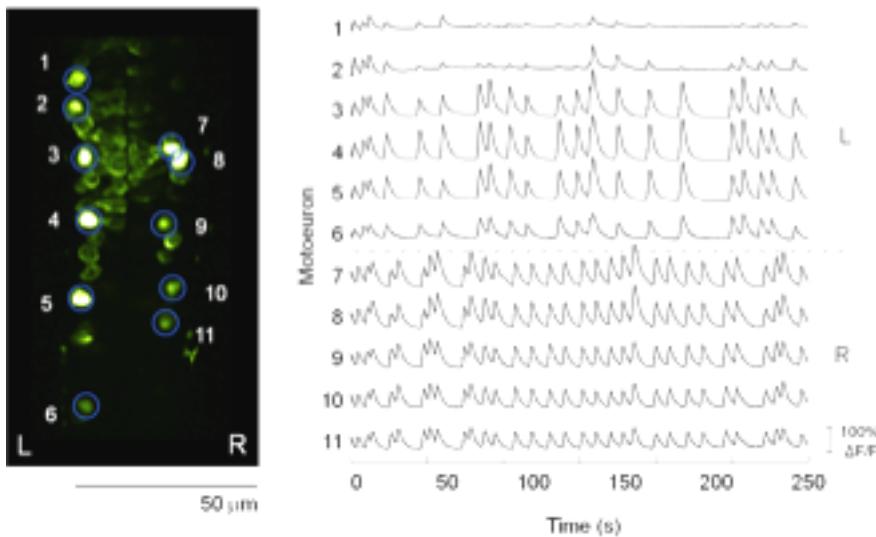


# Functional organization of motoneurons (Microscale)



5x

# Hierarchical node centrality

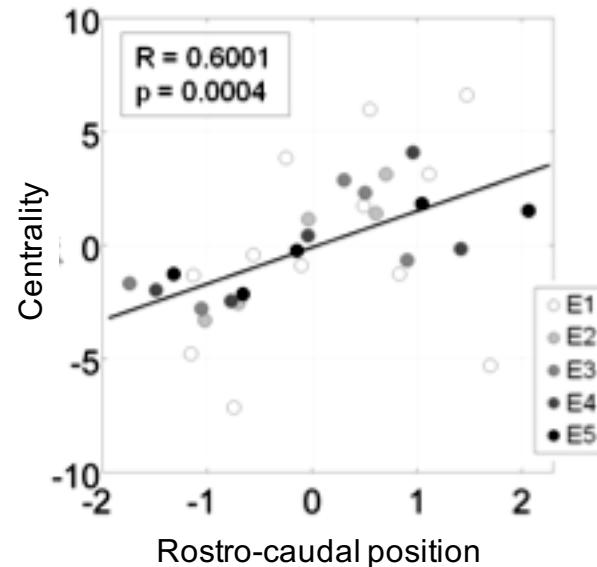
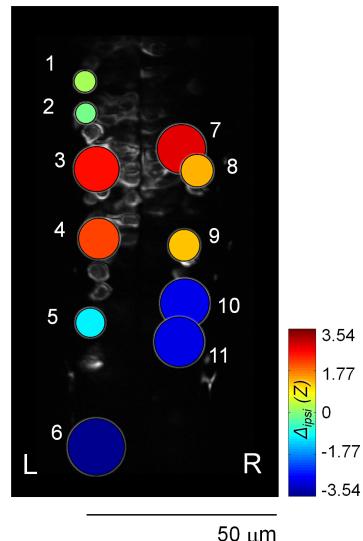


## Centrality

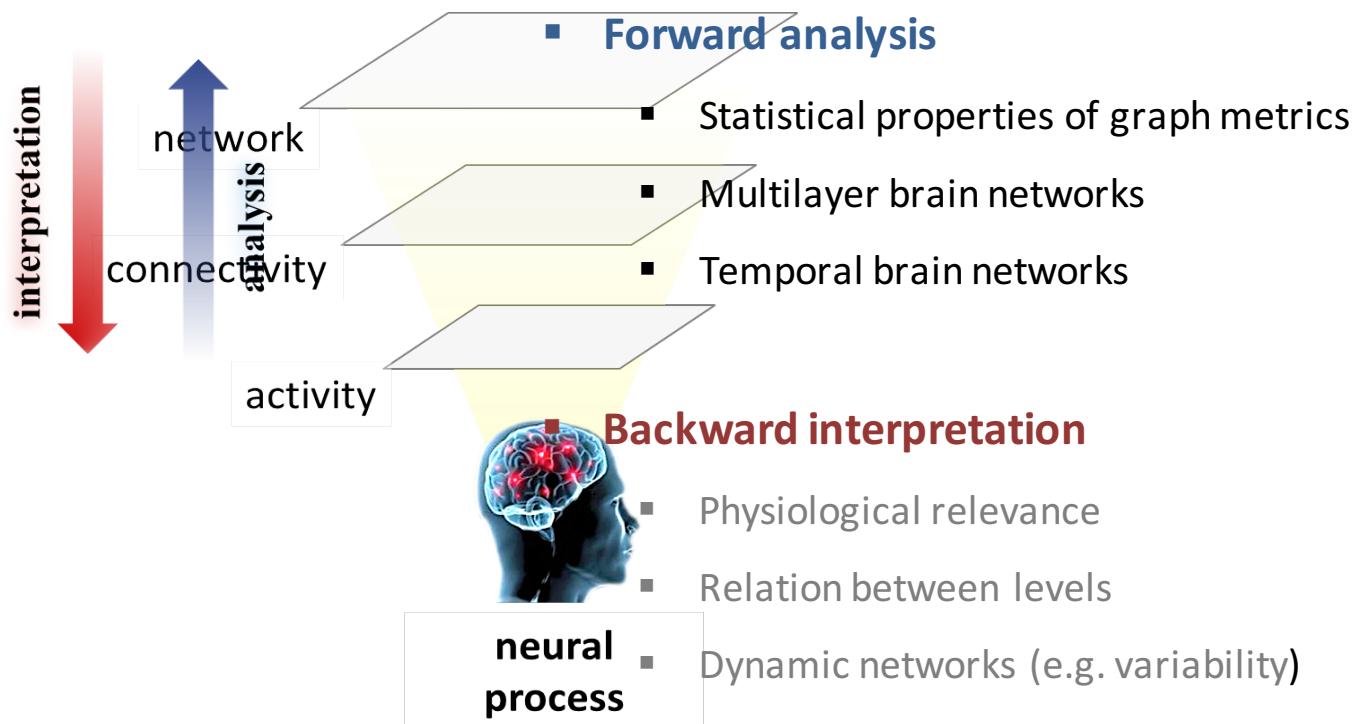
$$C(i) = k_{out}(i) - k_{in}(i)$$

$C(i) > 0 \rightarrow$  transmitter

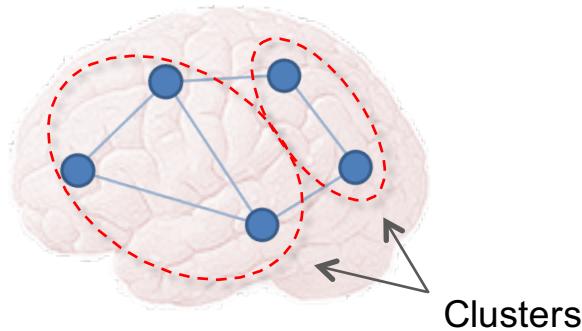
$C(i) < 0 \rightarrow$  receiver



# Some open issues



# Statistical methods for network clustering



$$\left\{ T_{i,1}^*, \dots, T_{i,N}^* \right\} \approx \text{Multinomial}(k_i; T_{i,1}, \dots, T_{i,N})$$

R bootstrap replicates (MC sampling)

$$\left\{ \mathbf{T}_1^*, \dots, \mathbf{T}_R^* \right\} \xrightarrow{\quad} \left\{ \mathbf{D}_1^*, \dots, \mathbf{D}_R^* \right\} \xrightarrow{\quad} \overline{\mathbf{D}}^* = \sum_{r=1}^R \mathbf{D}_r^*$$

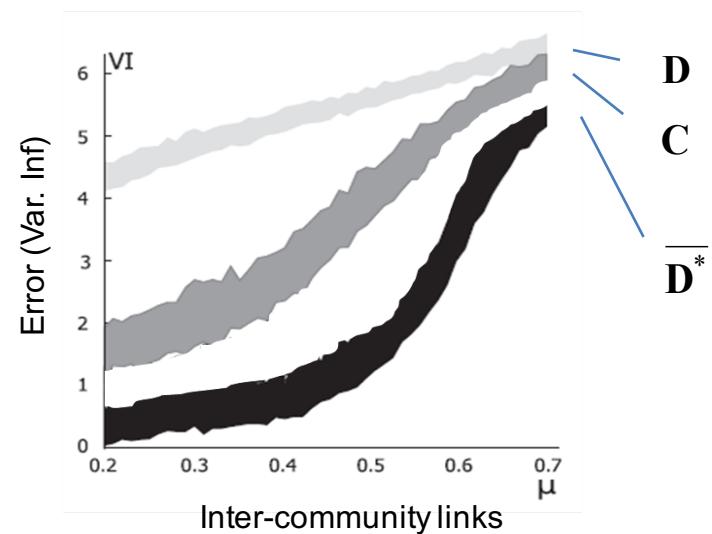
Transition matrix  $\mathbf{T}$

$$T_{i,j} = C_{i,j} / k_i \quad \leftarrow \text{Node degree}$$

Distance matrix  $\mathbf{D}$

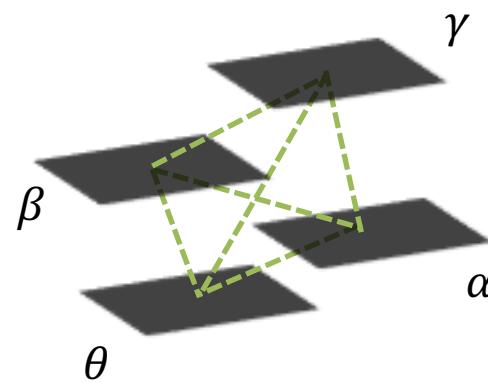
$$D_{i,j} \cong \sqrt{\sum_{l=1}^o \lambda_l^2 [v_l(i) - v_l(j)]^2}$$

**Synthetic networks** ( $N=500$ , 100 iter)



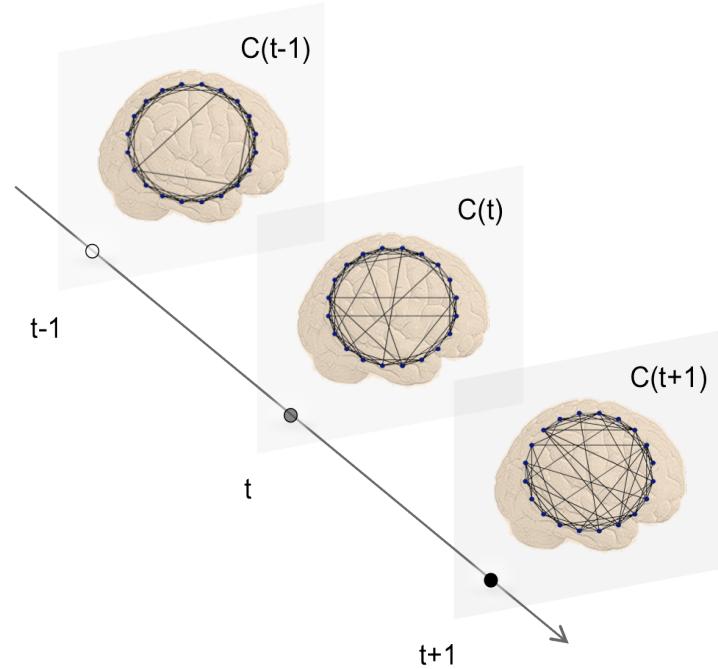
# Ongoing work

## Multilayer brain networks



Jeremy Guillon,  
UPMC Phd student

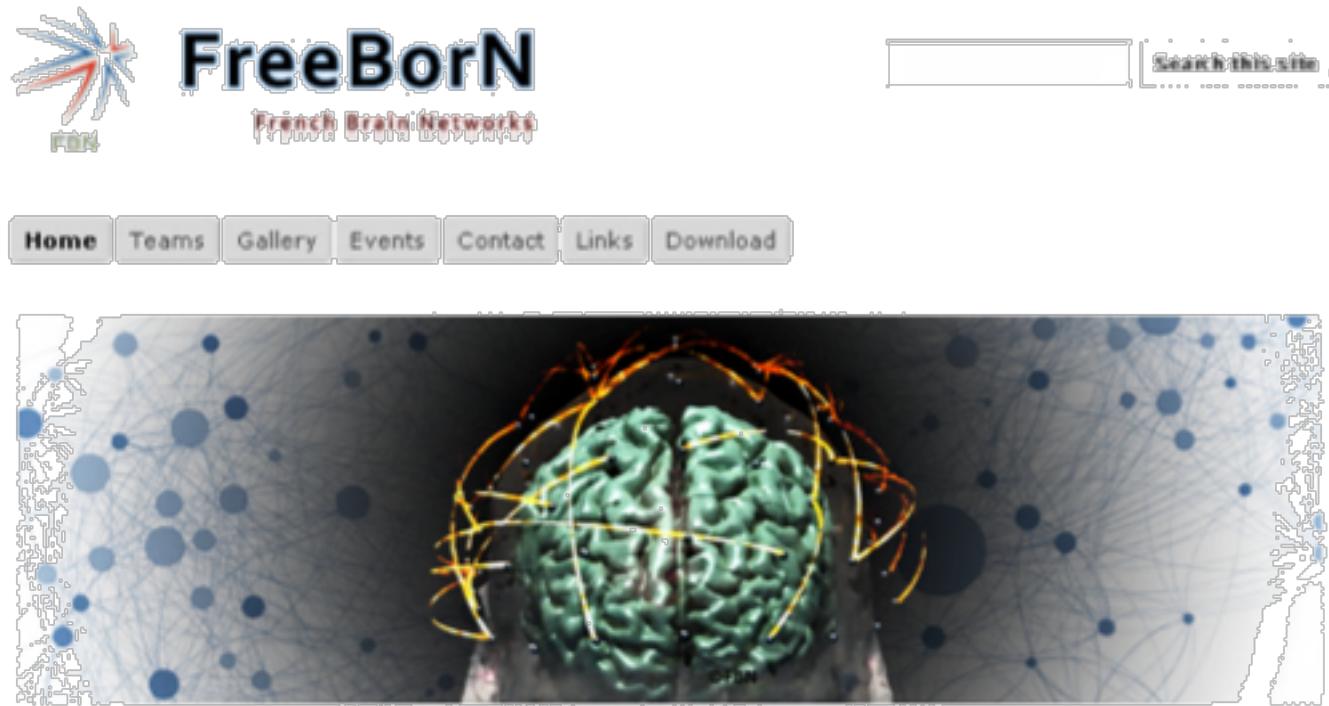
## Temporal brain networks



Catalina Obando,  
INRIA Phd student

Stay tuned ☺

<https://sites.google.com/site/fr2eborn/>



Articles, Routines, Data, Events, Job offers, ...

[@FreebornGroup](#)

# Acknowledgement

ARAMIS team ([www.aramislab.fr](http://www.aramislab.fr))



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- Marie Chupin (PI, CNRS - IR)
- Olivier Colliot (PI, CNRS)
- Stanley Durrleman (PI, INRIA)
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- Denis Schwartz (INSERM, Centre M/EEG)
- Laurent Hugheville (IR Centre EEG-MEG)
- ....

