Distributed Computing in Dynamic Networks

Impact of the dynamics on definitions and feasibility

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Distributed Computing



Collaboration of distinct entities to perform a common task.

No centralization available. Direct interaction.

(Think globally, act locally)

Broadcast

Propagating a piece of information from one node to all others.



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Election

Distinguishing exactly one node among all.



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Spanning tree

Selecting a cycle-free set of edges that interconnects all nodes.

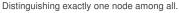


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Determining how many participants there are.

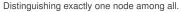


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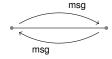
Counting

Determining how many participants there are.



Consensus, naming, routing, exploration, ...

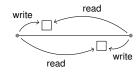




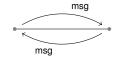
(a) Message passing



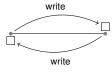
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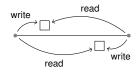
(b) Registers



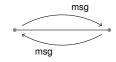
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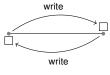
(c) Mailboxes



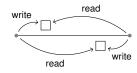
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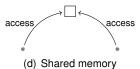
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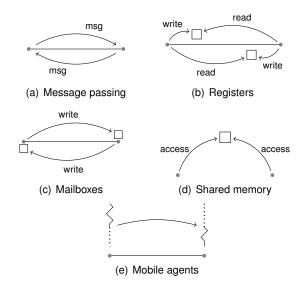


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(b) Registers





Atomic interaction

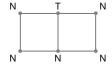
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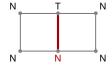
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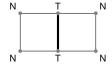
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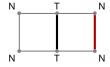
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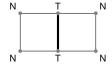
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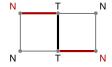
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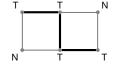
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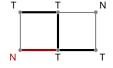
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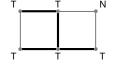
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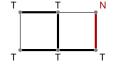
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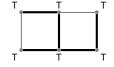
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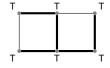
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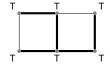
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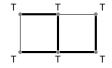
Relations between them (Chalopin, 2006)

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In fact, *highly* dynamic networks.













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How changes are perceived?

- Faults and Failures?
- Nature of the system. Change is normal.
- Partitioned network



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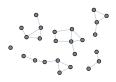






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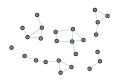




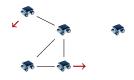


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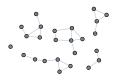




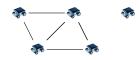


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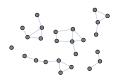






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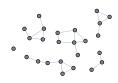






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(say, exploration by mobile robots)





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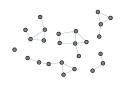






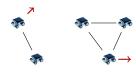
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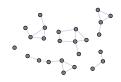






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Global point of view

Sequence of static graphs $\mathcal{G} = \textit{G}_0, \textit{G}_1, ...$ [+table of dates in \mathbb{T}]









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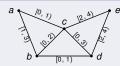


Local point of view

$$\mathcal{G} = (V, E, \mathcal{T}, \rho),$$

with ρ being a presence function

$$\rho: E \times \mathcal{T} \rightarrow \{0,1\}$$



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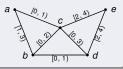


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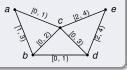


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- \rightarrow Further extensions possible (latency function, node-presence function, ...)

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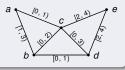


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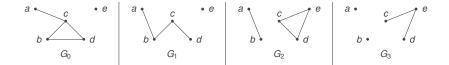
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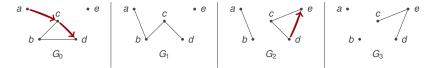
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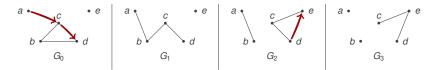
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For references, see (Ferreira, 2004) and (C., Flocchini, Quattrociocchi, Santoro, 2012)





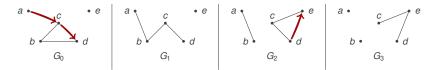
Ex:
$$((ac, t_1), (cd, t_2), (de, t_3))$$
 with $t_{i+1} \ge t_i$ and $\rho(e_i, t_i) = 1$



⇒ Paths become temporal (*journey*)

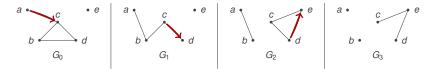
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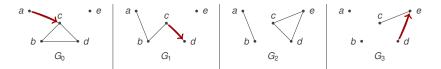
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- ⇒ *Strict* journeys *vs. non-strict* journeys. (Important for analysis.)



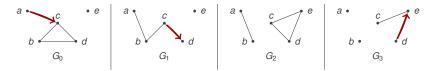
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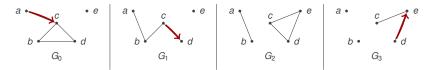


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In the literature: Schedule-conforming path (Berman, 1996); Time-respecting path (Kempe et al., 2008; Holme, 2005); Temporal path (Chaintreau et al., 2008); Journey (Bui-Xuan et al., 2003).



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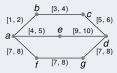
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Many other concepts... (ask me!).

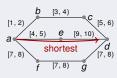




Broadcast?



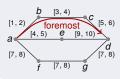
Broadcast?



Which way is optimal from *a* to *d*?

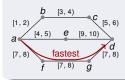
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Broadcast?



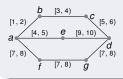
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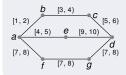
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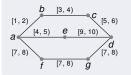
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Associated problems

 Computing shortest, foremost and fastest journeys in dynamic networks (centralized, with prior knowledge of the evolution, (Bui-Xuan, Jarry, Ferreira, 2003))

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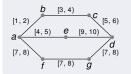
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Associated problems

- Computing shortest, foremost and fastest journeys in dynamic networks (centralized, with prior knowledge of the evolution, (Bui-Xuan, Jarry, Ferreira, 2003))
- Shortest, foremost and fastest broadcast in dynamic networks (distributed, with partial knowledge, (C., Flocchini, Mans, Santoro, 2012))

Broadcast?



Which way is optimal from a to d?

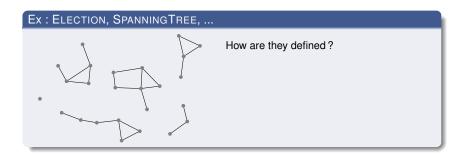
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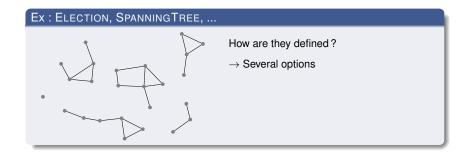
Associated problems

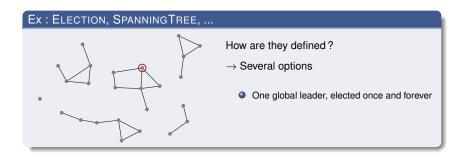
- Computing shortest, foremost and fastest journeys in dynamic networks (centralized, with prior knowledge of the evolution, (Bui-Xuan, Jarry, Ferreira, 2003))
- Shortest, foremost and fastest broadcast in dynamic networks (distributed, with partial knowledge, (C., Flocchini, Mans, Santoro, 2012))

Difficulty: Depends on the starting date \rightarrow set of time-dependent solutions.











How are they defined?

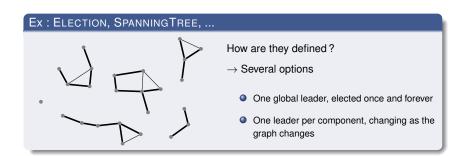
- \rightarrow Several options
 - One global leader, elected once and forever
 - One leader per component, changing as the graph changes

Ex: ELECTION, SPANNINGTREE, ...



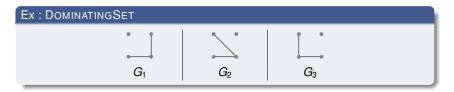
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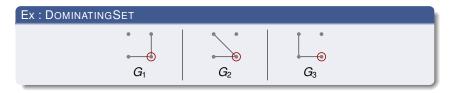


Both are very different in essence!





→ Temporal variant



→ Temporal variant

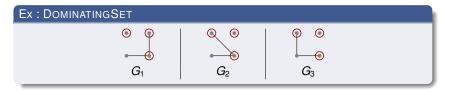


- → Temporal variant
- \rightarrow *Evolving* variant

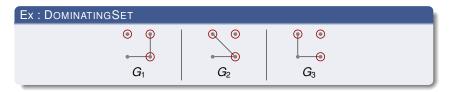
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Ex: DOMINATINGSET $G_1 \qquad G_2 \qquad G_3$

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Note : $PermanentDS \supseteq EvolvingDS_i \supseteq TemporalDS$.

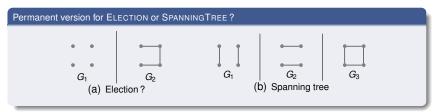
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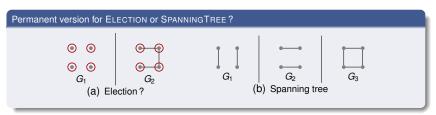
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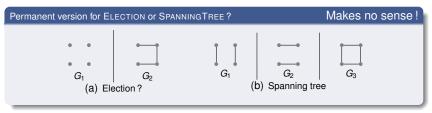
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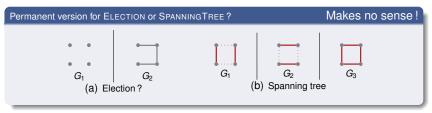
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Topological assumptions for distributed algorithms



Feasibility, Necessary and sufficient conditions, ...

Ex : Broadcast algorithm $\bullet \longrightarrow \bigcirc \longrightarrow \bigcirc$



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Lucky version.





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Lucky version. Yeah!!



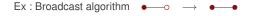


But things could have gone differently.





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But things could have gone differently. Too late!





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But things could have gone differently. Too late! Failure!

Ex : Broadcast algorithm $\bullet \longrightarrow \bigcirc \longrightarrow \bigcirc$



Or even worse..





Or even worse.. Too fast!





Or even worse.. Too fast! Too fast!





Or even worse.. Too fast! Too fast! Failure!



 \implies Additional assumptions needed to guarantee something.



→ Additional assumptions needed to guarantee something.

Assumption: Every present edge is "selected" at least once (but we don't know in what order...)



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Informal example

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Notions of *necessary condition* (e.g. $src \leadsto *$) or *sufficient condition* (e.g. $src \overset{st}{\leadsto} *$) for a given algorithm. These conditions relate only to the topology.



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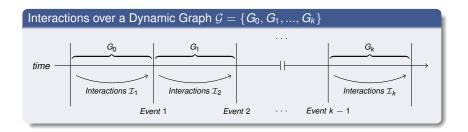
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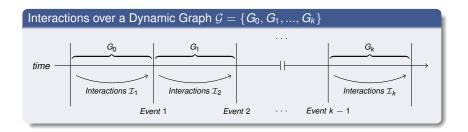
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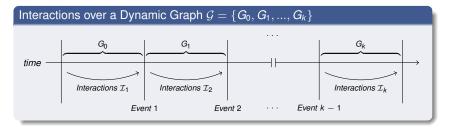
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More formally...



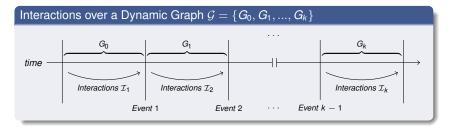






An execution is an alternated sequence of interactions and topological events :

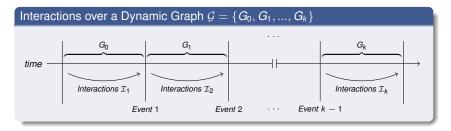
$$\textit{X} = \mathcal{I}_k \circ \textit{Event}_{k-1} \circ ... \circ \textit{Event}_2 \circ \mathcal{I}_2 \circ \textit{Event}_1 \circ \mathcal{I}_1(\textit{G}_0)$$



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Non deterministic!

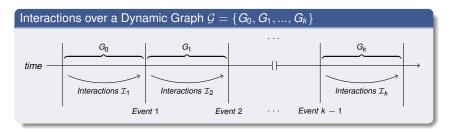


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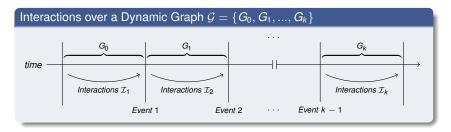
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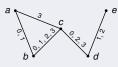
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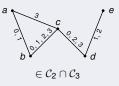
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 $\rightarrow \mathcal{C}_5$: at least one node verifies \mathcal{P} , (noted 1-*).

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- $\rightarrow \mathcal{C}_5$: at least one node verifies \mathcal{P} , (noted 1-*).
- $\rightarrow \mathcal{C}_6$: all the nodes verify \mathcal{P} , (noted *-*).

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• Necessary condition $\mathcal{C}_{\mathcal{N}}$: at least one node can be reached by all (* \leadsto 1).

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Conditions and classes of graphs

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 - $\rightarrow \mathcal{C}_7$: graphs having this property.
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 - $\rightarrow \mathcal{C}_6$ (already seen before).

Tightness of a condition?

(Marchand de Kerchove, Guinand, 2012)

Ned	cessary condition
Suf	ficient condition

Necessary condition

Not satisfied

failure is guaranteed

 $(\nexists X, success(X))$

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Remark: Topological conditions which are both necessary and sufficient may not exist!

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Remark: Topological conditions which are both necessary and sufficient may not exist!

Ex. uniform counting (last algorithm):

- → (*~1) is a tight necessary condition
- → (*-*) is a tight sufficient condition

In between: outcome is uncertain... might succeed or fail.

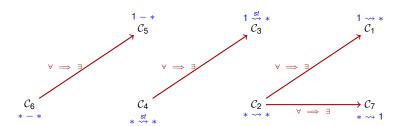


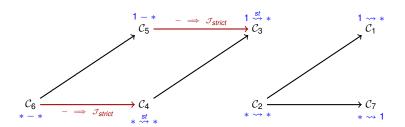


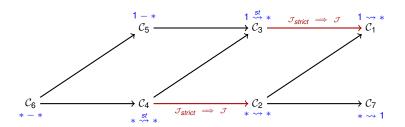
$$1 \xrightarrow{*} * \mathcal{C}_1$$

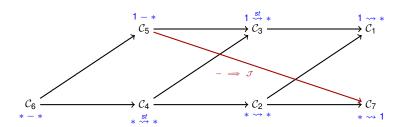


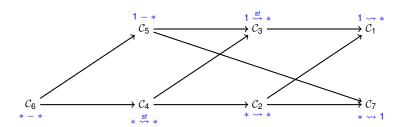
$$C_4$$
 $* \stackrel{st}{\leadsto} *$

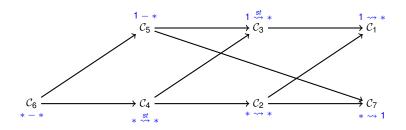




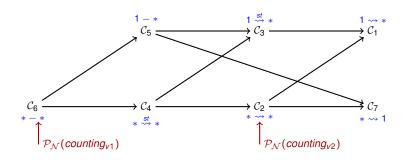




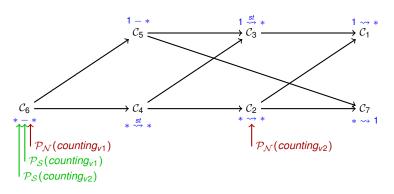




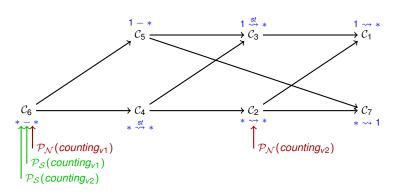
→ Comparison of algorithms on a formal basis



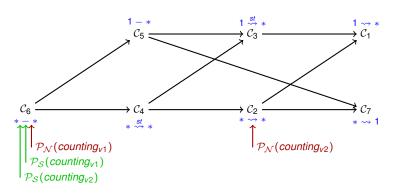
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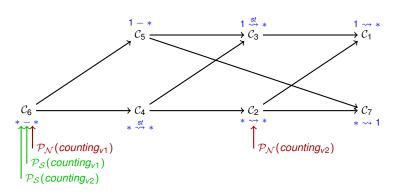
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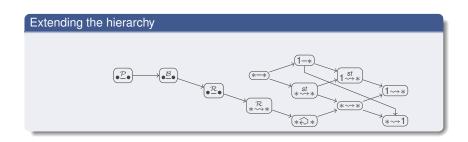
- → Comparison of algorithms on a formal basis
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- → Formal proofs ? (Coq)

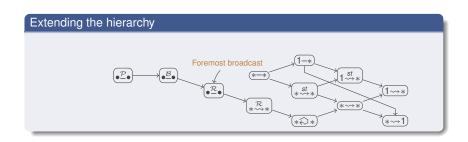


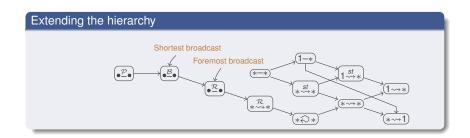
- → Comparison of algorithms on a formal basis
- \rightarrow Decision making (what algorithm to use?)
 - ightarrow e.g. using automated property checking on network traces).
- → Formal proofs ? (Coq)

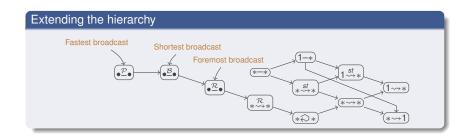
Q: How far beyond toy examples?

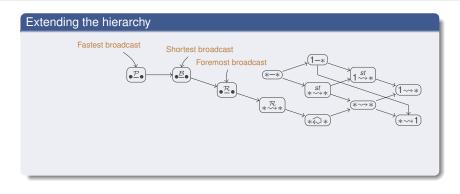




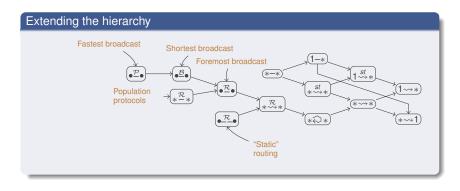


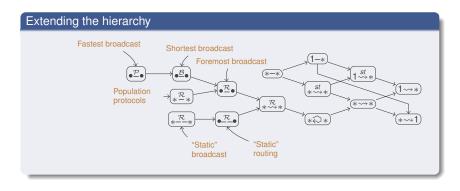


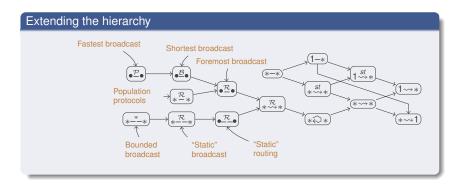




Extending the hierarchy Fastest broadcast Shortest broadcast 1-* Foremost broadcast <u>B</u>• 1 *** *-* R. * *** * * 1~~* Population protocols *** ** (*~→1)



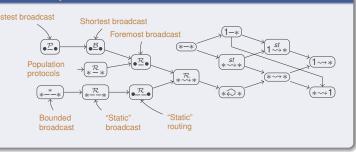


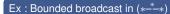


Extending the hierarchy Shortest broadcast Foremost broadcast 1 ×× s *-* Z. (1~→× Population $*_{*}^{\mathcal{R}}$ (*~~* *⇔* *~→1 Bounded routing broadcast broadcast



Extending the hierarchy





(O'Dell and Wattenhofer, 2005

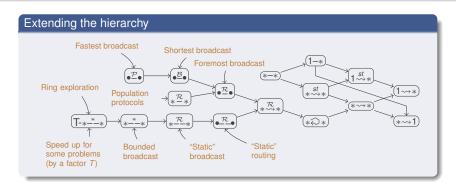
The graph is arbitrarily dynamic, as long as every G_i remains connected:

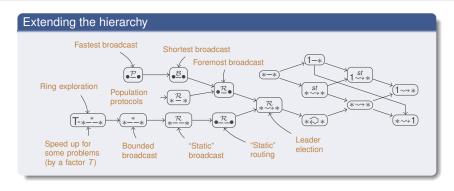


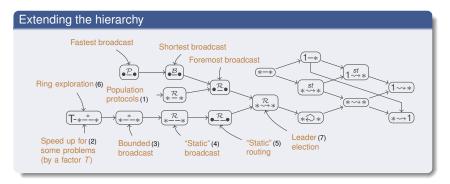
Min cut of size 1 between informed and uninformed nodes :

→ At least one new node informed in each step.



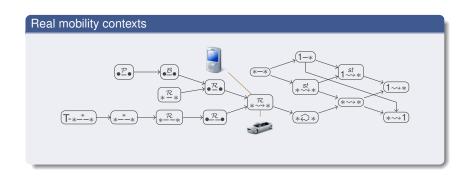


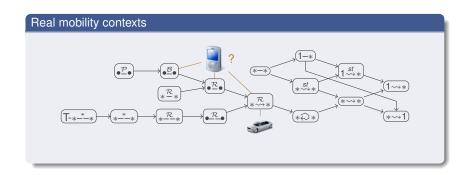


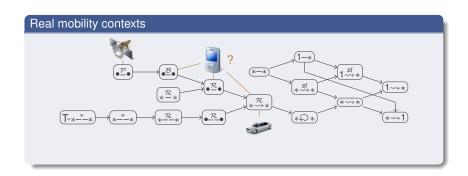


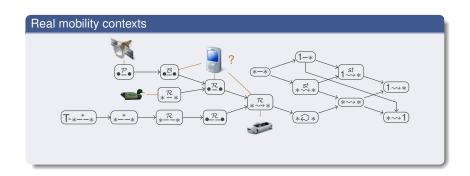
- Complete graph of interaction (Angluin, Aspnes, Diamadi, Fischer, Peralta, 2004)
- T-interval connectivity (Kuhn, Lynch, Oshman, 2010)
- Constant connectivity (O'Dell and Wattenhofer, 2005)
- Eventual instant connectivity (Ramanathan, Basu, and Krishnan, 2007)
- Eventual instant routability (Ramanathan, Basu, and Krishnan, 2007)
- T-interval connectivity (*Ilcinkas*, *Wade*, 2013)
- Recurrent temporal connectivity (Arantes, Greve, Sens, Simon, 2013) (Gòmez-Cazaldo, Lafuente, Larrea, Raynal, 2013)

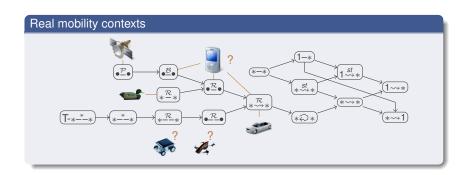
Real mobility contexts Performance of the contexts of the context of the context

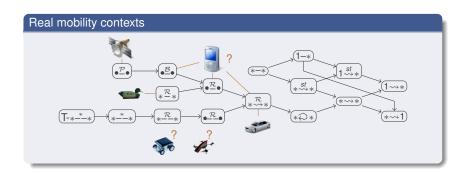






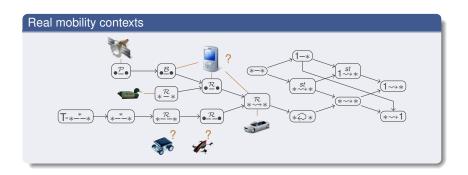






How to proceed?

- → Generate connection traces
 - → Test properties



How to proceed?

- → Generate connection traces
 - → Test properties
 - Temporal Connectivity (Whitbeck et al. 2012; Barjon et al., 2014)
 - T-Interval Connectivity (C. et al., 2014)

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