The Random Subgraph Model for the Analysis of an Ecclesiastical Network in Merovingian Gaul

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This is a joint work with
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Introduction

The random subgraph model (RSM)

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Numerical experiments

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Conclusion

Introduction

The analysis of networks:

- is a recent but increasingly important field in statistical learning,
- with applications in domains ranging from biology to history:
  - biology: analysis of gene regulation processes,
  - social sciences: analysis of political blogs,
  - history: visualization of medieval social networks.

Two main problems are currently well addressed:

- visualization of the networks,
- clustering of the network nodes.

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Network comparison:

- is a still emerging problem is statistical learning,
- which is mainly addressed using graph structure comparison,
- but limited to binary networks.

Introduction

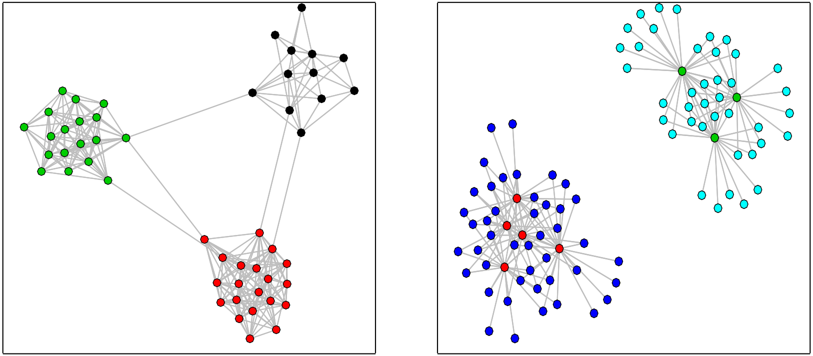

Figure : Clustering of network nodes: communities (left) *vs.* structures with hubs (right).

Introduction

Key works in probabilistic models:

- stochastic block model (SBM) by Nowicki and Snijders (2001),
- latent space model by Hoff, Handcock and Raftery (2002),
- latent cluster model by Handcock, Raftery and Tantrum (2007),
- mixed membership SBM (MMSBM) by Airoldi et al. (2008),
- mixture of experts for LCM by Gormley and Murphy (2010),
- MMSBM for dynamic networks by Xing et al. (2010),
- overlapping SBM (OSBM) by Latouche et al. (2011).

A good overview is given in:

- M. Salter-Townshend, A. White, I. Gollini and T. B. Murphy, "Review of Statistical Network Analysis: Models, Algorithms, and Software", Statistical Analysis and Data Mining, Vol. 5(4), pp. 243–264, 2012.

Introduction: the historical problem

Our colleagues from the LAMOP team were interested in answering the following question:

Does the Church was organized in the same way within the different kingdoms in Merovingian Gaul?

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Does the Church was organized in the same way within the different kingdoms in Merovingian Gaul?

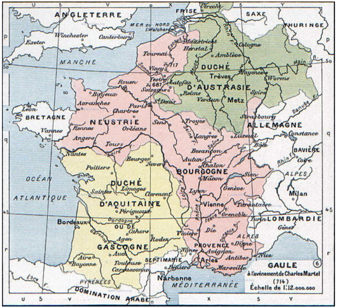
To this end, they have build a relational database:

- from written acts of ecclesiastical councils that took place in Gaul during the 6th century (480-614),
- those acts report who attended (bishops, kings, dukes, priests, monks, ...) and what questions (regarding Church, faith, ...) were discussed,
- they also allowed to characterize the type of relationship between the individuals,
- it took 18 months to build the database.

Introduction: the historical problem

The database contains:

- 1331 individuals (mostly clergymen) who participated to ecclesiastical councils in Gaul between 480 and 614,
- 4 types of relationships between individuals have been identified (positive, negative, variable or neutral),
- each individual belongs to one of the 5 regions of Gaul:
  - 3 kingdoms: Austrasia, Burgundy and Neustria,
  - 2 provinces: Aquitaine and Provence.



- additional information is also available: *social positions*, family relationships, birth and death dates, hold offices, councils dates, ...

Introduction: the historical problem

Figure : Adjacency matrix of the ecclesiastical network (sorted by regions).

Introduction

Expected difficulties:

- existing approaches can not analyze networks with categorical edges and a partition into subgraphs,
- comparison of subgraphs has, up to our knowledge, not been addressed in this context,
- a "source effect" is expected due to the overrepresentation of some places (Neustria through "Ten History Book" of Gregory of Tours) or individuals (hagiographies).

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Our approach:

- we consider directed networks with typed (categorical) edges and for which a partition into subgraphs is known,
- we base our comparison on the cluster organization of the subgraphs,
- we propose an extension of SBM which takes into account typed edges and subgraphs,
- subgraph comparison is possible afterward using model parameters.

Plan de l'exposé

The random subgraph model (RSM)

Before the maths, an example of an RSM network:

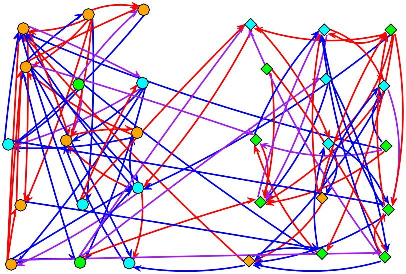

Figure : Example of an RSM network.

We observe:

- the partition of the network into $S = 2$ subgraphs (node form),
- the presence A_{ij} of directed edges between the N nodes,
- the type $X_{ij} \in \{1, ..., C\}$ of the edges ($C = 3$, edge color).

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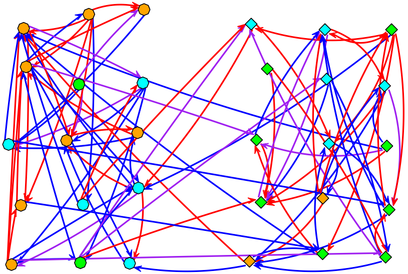

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- the presence A_{ij} of directed edges between the N nodes,
- the type $X_{ij} \in \{1, ..., C\}$ of the edges ($C = 3$, edge color).

We search:

- a partition of the node into $K = 3$ groups (node color),
- which overlap with the partition into subgraphs.

The random subgraph model (RSM)

The network (represented by its adjacency matrix X) is assumed to be generated as follows:

- the presence of an edge between nodes i and j is such that:

$$A_{ij} \sim \mathcal{B}(\gamma_{s_i s_j})$$

where $s_i \in \{1, ..., S\}$ indicates the (observed) subgraph of node i,

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  where $s_i \in \{1, ..., S\}$ indicates the (observed) subgraph of node i,
- each node i is as well associated with an (unobserved) group among K according to:

$$Z_i \sim \mathcal{M}(\alpha_{s_i})$$

  where $\alpha_s \in [0, 1]^K$ and $\sum_{k=1}^{K} \alpha_{sk} = 1$,

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  where $\alpha_s \in [0, 1]^K$ and $\sum_{k=1}^{K} \alpha_{sk} = 1$,
- each edge X_{ij} can be finally of C different (observed) types and such that:

$$X_{ij} | A_{ij} Z_{ik} Z_{jl} = 1 \sim \mathcal{M}(\Pi_{kl})$$

  where $\Pi_{kl} \in [0, 1]^C$ and $\sum_{c=1}^{C} \Pi_{klc} = 1$.

The random subgraph model (RSM)

Notations	Description
$\mathbf{X}$	Adjacency matrix. $X_{ij} \in \{0, \ldots, C\}$ indicates the edge type
$\mathbf{A}$	Binary matrix. $A_{ij} = 1$ indicates the presence of an edge
$\mathbf{Z}$	Binary matrix. $Z_{ik} = 1$ indicates that i belongs to cluster k
N	Number of vertices in the network
K	Number of latent clusters
S	Number of subgraphs
C	Number of edge types
$\boldsymbol{\alpha}$	α_{sk} is the proportion of cluster k in subgraph s
$\boldsymbol{\Pi}$	Π_{klc} is the probability of having an edge of type c between vertices of clusters k and l
$\boldsymbol{\gamma}$	γ_{rs} probability of having an edge between vertices of subgraphs r and s

Table : Summary of the notations.

The random subgraph model (RSM)

Remark 1:

- the RSM model separates the roles of the known partition and the latent clusters,
- this was motivated by historical assumptions on the creation of relationships during the 6th century,
- indeed, the possibilities of connection were preponderant over the type of connection and mainly dependent on the geography.

The random subgraph model (RSM)

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Remark 2:

- an alternative approach would consist in allowing X_{ij} to directly depend on both the latent clusters and the partition,
- however, this would dramatically increase the number of model parameters ($K^2S^2(C+1)+SK$ instead of S^2+K^2C+SK),
- if $S=6$, $K=6$ and $C=4$, then the alternative approach has 6 516 parameters while RSM has only 216.

The random subgraph model (RSM)

We consider a Bayesian framework:

- the previous model is fully defined by its joint distribution:

$$p(X, A, Z|\alpha, \gamma, \Pi) = p(X|A, Z, \Pi)p(A|\gamma)p(Z|\alpha),$$

- which we complete with conjuguate prior distributions for model parameters:
  - the prior distribution for α is:

$$p(\gamma_{rs}) = Beta(a_{rs}, b_{rs}),$$

  - the prior distribution for γ is:

$$p(\alpha_s) = Dir(\chi_s),$$

  - the prior distribution for Π is:

$$p(\Pi_{kl}) = Dir(\Xi_{kl}).$$

The random subgraph model (RSM)

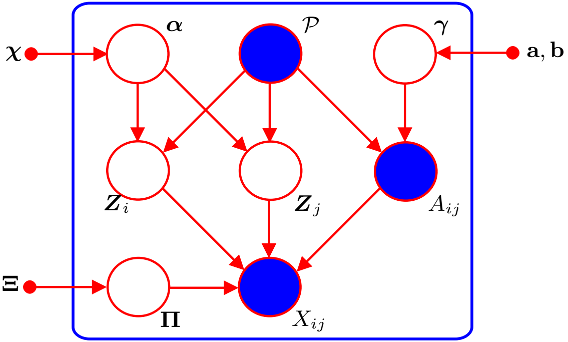

Figure : A graphical representation of the RSM model.

Plan de l'exposé

Model inference

Due to the Bayesian framework introduces above:

- we aim at estimating the posterior distribution $p(Z, \alpha, \gamma, \Pi | X, A)$, which in turn will allow us to compute MAP estimates of Z and (α, γ, Π),
- as expected, this distribution is not tractable and approximate inference procedures are required,
- the use of MCMC methods is obviously an option but MCMC methods have a poor scaling with sample sizes.

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We chose to use variational approaches:

- because they allow to deal with large networks $(N > 1000)$,
- recent theoretical results (Celisse et al., 2012; Mariadassou and Matias, 2013) gave new insights about convergence properties of variational approaches in this context.

The EM, VEM and VBEM algorithms

First, it necessary to write the log-likelihood as:

$$\log(p(X|\theta)) = \mathcal{L}(q(Z);\theta) + KL(q(Z)||p(Z|X,\theta)),$$

where:

- $\mathcal{L}(q(Z);\theta) = \sum_Z q(Z)\log(p(X,Z|\theta)/q(Z))$ is a lower bound of the log-likelihood,
- $KL(q(Z)||p(Z|X,\theta)) = -\sum_Z q(Z)\log(p(X|Z,\theta)/q(Z))$ is the KL divergence between $q(Z)$ and $p(Z|X,\theta)$.

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The EM algorithm:

- E step: θ is fixed and $\mathcal{L}$ is maximized over $q \Rightarrow q^*(Z) = p(Z|X,\theta)$
- M step: $\mathcal{L}(q^*(Z),\theta^{old})$ is now maximized over θ

$$\begin{aligned}\mathcal{L}(q^*(Z),\theta^{old}) &= \sum_Z p(Z|X,\theta^{old})\log(p(X,Z|\theta)/p(Z|X,\theta^{old}))\\ &= E[\log(p(X,Z|\theta)|\theta^{old}] + c.\end{aligned}$$

The EM, VEM and VBEM algorithms

The variational approach:

- let us now suppose that $p(X, Z|\theta)$ is, for some reason, intractable,
- the variational approach restrict the range of functions for q such that the problem is tractable,
- a popular variational approximation is to assume that q factorizes:

$$q(Z) = \prod_i q_i(Z_i).$$

The VEM algorithm:

- V-E step: θ is fixed and $\mathcal{L}$ is maximized over $q \Rightarrow$ $\log q_j^*(Z_j) = E_{i \neq j}[\log p(X, Z|\theta)] + c$
- V-M step: $\mathcal{L}(q^*(Z), \theta^{old})$ is now maximized over θ

The EM, VEM and VBEM algorithms

We consider now the Bayesian framework:

- we aim at estimating the posterior distribution $p(Z, \theta|X)$,
- we have here the relation:

$$\log(p(X)) = \mathcal{L}(q(Z, \theta)) + KL(q(Z, \theta)||p(Z, \theta|X)),$$

- we also assume that q factorizes over Z and θ:

$$q(Z, \theta) = \prod_i q_i(Z_i) q_\theta(\theta).$$

The VBEM algorithm:

- VB-E step: $q_\theta(\theta)$ is fixed and $\mathcal{L}$ is maximized over the q_i $\Rightarrow$ $\log q_j^*(Z_j) = E_{i \neq j, \theta}[\log p(X, Z, \theta)] + c$
- VB-M step: all $q_i(Z_i)$ are now fixed and $\mathcal{L}$ is maximized over q_θ $\Rightarrow$ $\log q_\theta^*(\theta) = E_Z[\log p(X, Z, \theta)] + c$

The VBEM algorithm for RSM

Variational Bayesian inference in our case:

- we aim at approximating the posterior distribution $p(Z, \alpha, \gamma, \Pi | X, A)$
- we therefore search the approximation $q(Z, \alpha, \gamma, \Pi)$ which maximizes $\mathcal{L}(q)$ where:

$$\log p(X, A) = \mathcal{L}(q) + KL(q || p(.|X, A)),$$

- and q is assumed to factorize as follows:

$$q(Z, \alpha, \gamma, \Pi) = \prod q(Z_i) \prod q(\alpha_s) \prod q(\gamma_{st}) \prod q(\Pi_{kl}).$$

The VBEM algorithm for RSM:

- E step: compute the update parameter τ_i for $q(Z_i)$,
- M step: compute the update parameters χ, γ, Ξ for respectively $q(\alpha_s)$, $q(\gamma_{st})$ and $q(\Pi_{kl})$.

The VBEM algorithm for RSM: the M step

The M step of the VBEM algorithm: the VBEM update step for the distributions $q(\alpha_s)$ is:

$$\begin{aligned}\log q^*(\alpha_s) &= E_{Z,\alpha^{\setminus s},\gamma,\Pi}[\log p(X, A, Z, \alpha, \gamma, \Pi)] + c \\ &= \sum_{k=1}^{K} \log(\alpha_{sk}) \left\{ \chi^0_{sk} + \sum_{i=1}^{N} \delta(r_i = s)\tau_{ik} - 1 \right\} + c,\end{aligned}$$

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which is the functional form for a Dirichlet distribution:

$$q(\alpha_s) = \mathrm{Dir}(\alpha_s; \chi_s), \forall s \in \{1, \ldots, S\}$$

where $\chi_{sk} = \chi_{sk}^0 + \sum_{i=1}^{N} \delta(r_i = s)\tau_{ik}, \forall k \in \{1, \ldots, K\}$.

The VBEM algorithm for RSM: the M step

The M step of the VBEM algorithm: the VBEM update step for the distributions $q(\alpha_s)$, $q(\gamma_{st})$ and $q(\Pi_{kl})$ are:

- $q(\alpha_s) = \mathrm{Dir}(\alpha_s; \chi_s), \forall s \in \{1, \dots, S\}$,
- $q(\gamma_{rs}) = \mathrm{Beta}(\gamma_{rs}; a_{rs}, b_{rs}), \forall (r, s) \in \{1, \dots, S\}^2$,
- $q(\Pi_{kl}) = \mathrm{Dir}(\Pi_{kl}; \Xi_{kl}), \forall (k, l) \in \{1, \dots, K\}^2$,

where:

- $\chi_{sk} = \chi^0_{sk} + \sum_{i=1}^{N} \delta(r_i = s)\tau_{ik}, \forall k \in \{1, \dots, K\}$,
- $a_{rs} = a^0_{rs} + \sum_{r_i = r, r_j = s}(A_{ij}),\ b_{rs} = b^0_{rs} + \sum_{r_i = r, r_j = s}(1 - A_{ij})$,
- $\Xi_{klc} = \Xi^0_{klc} + \sum_{i \neq j}^{N} \delta(X_{ij} = c)\tau_{ik}\tau_{jl}, \forall c \in \{1, \dots, C\}$.

The VBEM algorithm for RSM: the E step

The E step of the VBEM algorithm: the VBEM update step for the distribution $q(Z_i)$ is given by:

$$\log q^*(Z_i) = E_{Z^{\setminus i},\alpha,\gamma,\Pi}[\log p(X, A, Z, \alpha, \gamma, \Pi)] + c$$

which implies that

$$q(Z_i) = \mathcal{M}(Z_i; 1, \tau_i), \, \forall i = 1, ..., N$$

where

$$\begin{aligned}
\tau_{ik} \propto \exp & \left(\psi(\chi_{r_i,k}) - \psi(\sum_{l=1}^{K} \chi_{r_i,l}) \right) \\
& + \exp \left\{ \sum_{j \neq i}^{N} \sum_{c=1}^{C} \sum_{l=1}^{K} \delta(X_{ij} = c) \tau_{jl} \left(\psi(\Xi_{klc}) - \psi(\sum_{u=1}^{C} \Xi_{klu}) \right) \right\} \\
& + \exp \left\{ \sum_{j \neq i}^{N} \sum_{c=1}^{C} \sum_{l=1}^{K} \delta(X_{ji} = c) \tau_{jl} \left(\psi(\Xi_{lkc}) - \psi(\sum_{u=1}^{C} \Xi_{lku}) \right) \right\}.
\end{aligned}$$

Initialization and choice of K

Initialization of the VBEM algorithm:

- the VBEM is known to be sensitive to its initialization,
- we propose a strategy based on several k-means algorithms with a specific distance:

$$d(i,j) = \sum_{h=1}^{N} \delta(X_{ih} \neq X_{jh}) A_{ih} A_{jh} + \sum_{h=1}^{N} \delta(X_{hi} \neq X_{hj}) A_{hi} A_{hj}.$$

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Choice of the number K of groups:

- once the VBEM algorithm has converged, the lower bound $\mathcal{L}(q)$ is a good approximation of the integrated log-likelihood $\log p(X, A)$,
- we thus can use $\mathcal{L}(q)$ as a model selection criterion for choosing K,
- if computed right after the M step,

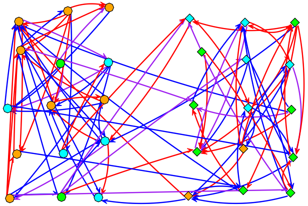
$$\mathcal{L}(q) = \sum_{r,s}^{S} \log\left(\frac{B(a_{rs}, b_{rs})}{B(a_{rs}^0, b_{rs}^0)}\right) + \sum_{s=1}^{S} \log\left(\frac{C(\boldsymbol{\chi}_s)}{C(\boldsymbol{\chi}_s^0)}\right) + \sum_{k,l}^{K} \log\left(\frac{C(\boldsymbol{\Xi}_{kl})}{C(\boldsymbol{\Xi}_{kl}^0)}\right) - \sum_{i=1}^{N} \sum_{k=1}^{K} \tau_{ik} \log(\tau_{ik}).$$

Plan de l'exposé

Experimental setup

We considered 3 different situations:

- S1 : network without subgraphs and with a preponderant proportion of edges of type 1,
- S2 : network without subgraphs and with balanced proportions of the three edge types,
- S3 : network with 3 subgraphs and with balanced proportions of the three edge types.



Global setup:

- in all cases, the number of (unobserved) groups is $K = 3$ and the network size is $N = 100$,
- we use the adjusted Rand index (ARI) for evaluating the clustering quality (and thus the model fitting).

Choice of the number K of groups

First, a model selection study:

- we aim at validating the use of $\mathcal{L}(q)$ as model selection criteria,
- we simulated 50 RSM networks according to scenario 1 and with $N = 100$,
- and applied our VB-EM algorithm for different values of K ($K = 2, ..., 5$),
- the actual value of K is $K = 3$.

Choice of the number K of groups

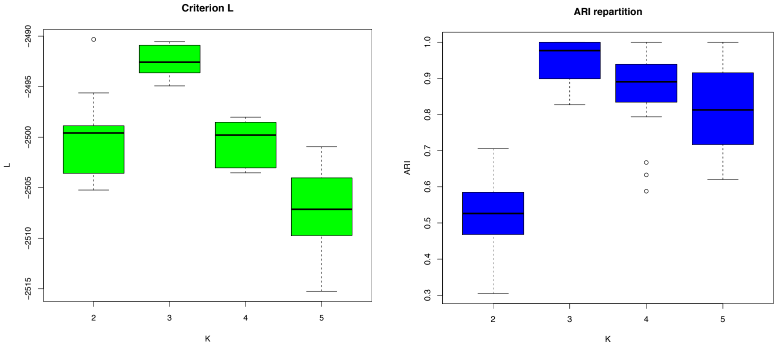


Table : Lower bound $\mathcal{L}$ and ARI averaged over 50 networks simulated according to the RSM model.

Comparison with other SBM-based approaches

Second, a comparison with other SBM-based methods:

- binary SBM: the original SBM algorithm was applied on a collapsed version of the data (only the presence of edges); the mixer package was used,
- binary SBM (type 1, 2 or 3): the original SBM algorithm was applied on a collapsed version of the data (only edges of type 1, 2 or 3); the mixer package was used,
- typed SBM: we had to implement the categorical version of SBM since it is not available in existing software; this version of SBM will be available in mixer soon,
- the studied methods were applied to the the three scenarii and results are averaged over 50 networks.

Comparison with other SBM-based approaches

Method	Scenario 1	Scenario 2	Scenario 3
binary SBM (presence)	0.001 ± 0.012	0.001 ± 0.013	0.239 ± 0.061
binary SBM (type 1)	0.976 ± 0.071	0.494 ± 0.233	-0.372 ± 0.262
binary SBM (type 2)	0.001 ± 0.006	-0.003 ± 0.006	0.179 ± 0.097
binary SBM (type 3)	0.959 ± 0.121	0.519 ± 0.219	0.367 ± 0.244
Typed SBM	0.694 ± 0.232	0.472 ± 0.339	0.360 ± 0.162
RSM	$\mathbf{1.000 \pm 0.000}$	$\mathbf{0.981 \pm 0.056}$	$\mathbf{0.939 \pm 0.097}$

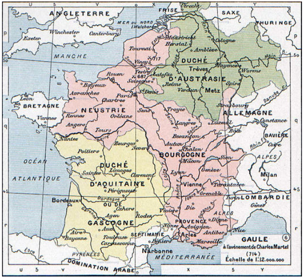
Table : ARI averaged over 50 networks simulated according to the three considered situations.

Plan de l'exposé

The ecclesiastical network

The data:

- 1331 individuals (mostly clergymen) who participated to ecclesiastical councils in Gaul between 480 and 614,
- 4 types of relationships between individuals have been identified (positive, negative, variable or neutral),
- each individual belongs to one of the 5 regions (3 kingdoms et 2 provinces).



Our modeling allows a multi-level analysis:

- Z allows to characterize the found clusters through social positions of the individuals,
- parameter Π describes the relations between the found clusters,
- parameter γ describes the connections between the subgraphs,
- parameter α describes the cluster repartition in the subgraphs.

RSM results: the latent clusters

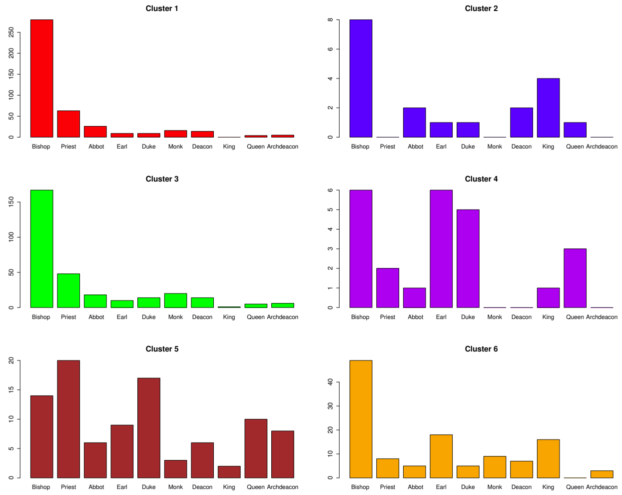


Figure : Characterization of the $K = 6$ clusters found by RSM.

RSM results: the latent clusters

The latent clusters from the historical point of view:

- clusters 1 and 3 correspond to local, provincial of diocesan councils, mostly interested in local issues (ex: council of Arles, 554),
- clusters 2 and 6 correspond to councils dedicated to political questions, usually convened by a king (ex: Orleans, 511),
- clusters 4 and 5 correspond to aristocratic assemblies, where queens and duke and earls are present (ex: Orleans, 529).

RSM results: the relationships between clusters

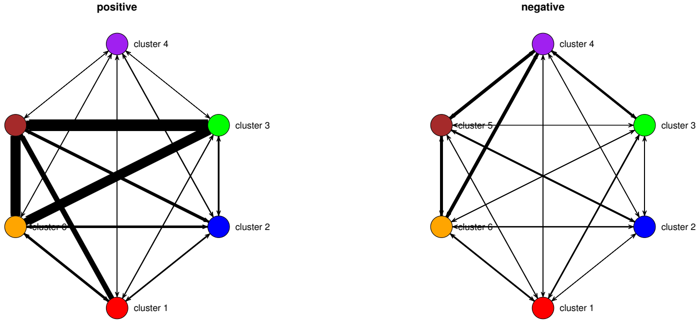
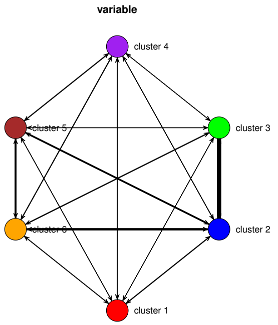


Figure : Characterization of the relationships between clusters (parameter Π).

RSM results: the relationships between clusters



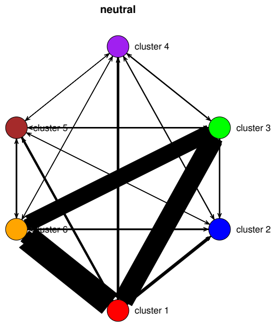


Figure : Characterization of the relationships between clusters (parameter Π).

RSM results: the relationships between clusters

The clusters relationships from the historical point of view:

- positive relations between clusters 3, 5 and 6 mainly corresponds to personal friendships between bishops (source effect),
- negative and variable relations betweens clusters 4, 5 and 6 report the conflicts in the hierarchy of the power,
- neutral relations between clusters 1, 3 and 6 were expected because they deal with different issues (local / political).

RSM results: the relationships between regions

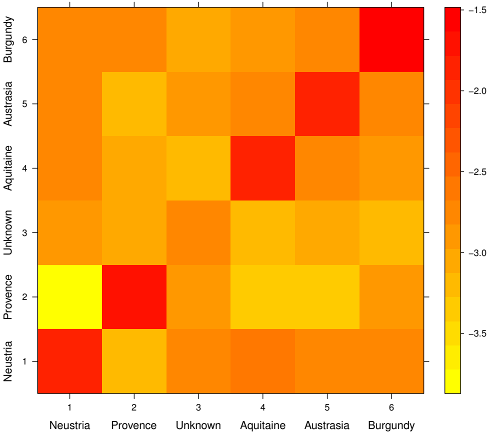


Figure : Characterization of the relationships between the regions (parameter γ in log scale).

RSM results: comparison of the regions

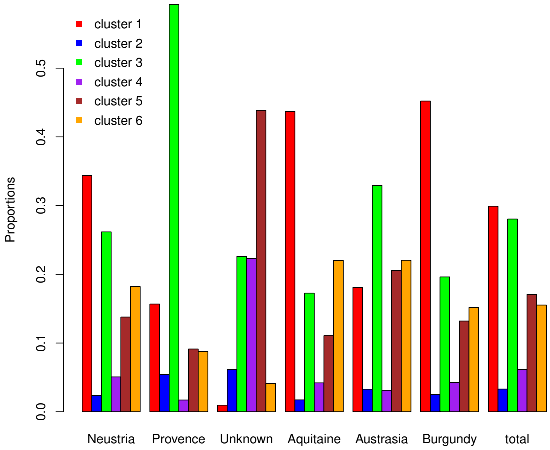


Figure : Characterization of regions through cluster repartition (parameter α).

RSM results: comparison of the regions

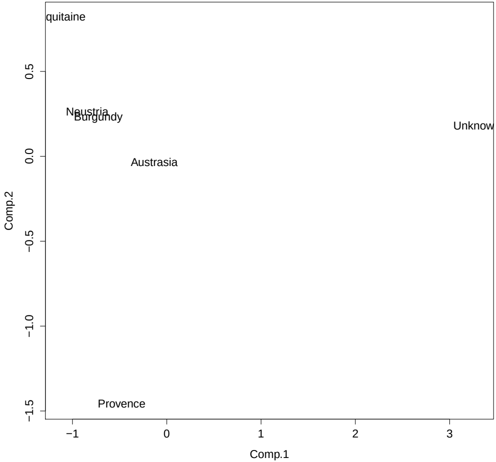


Figure : PCA for compositional data on the parameter α.

Plan de l'exposé

Conclusion

Our contribution:

- a model for network clustering which takes into account an existing partition of the network into subgraphs,
- this modeling allows afterward a comparison of the subgraphs,
- inference is done in a Bayesian framework using a VBEM algorithm,
- our approach has been applied to a complex historical network.

Software:

package Rambo for the R software is available on the CRAN

Preprint:

`http://arxiv.org/abs/1212.5497`

Further work

Interesting problems to address:

- discrete edges which are frequent in many applications,
- temporality of the network (evolution of relations, offices or social positions),
- visualization of this kind of networks,
- procedures to test the similarity of subgraphs ...

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One Ph.D. position is available on this topic!